Let \( \Omega \subseteq \mathbb{R}^d \), \( d \geq 1 \), be a bounded domain with boundary \( \Gamma \). Denote by \( X_{\Omega,t} \) the Lebesgue measure on \( \Omega \), by \( \sigma_{\Omega,t} \) the surface measure on \( \Gamma \) and let \( \alpha \in C^2(\Gamma) \), \( \alpha > 0 \).

Then the solution of the following SDE describes a distorted Brownian motion in \( \Omega \) with (immediate) reflection at \( \Gamma \):

\[
dX_t = dB_t + \frac{\nabla \phi(x)}{\left| \nabla \phi(x) \right|} \, dt, \quad x \in \Omega.
\]

where \( (B_t)_{t \geq 0} \) is an \( \mathbb{R}^d \)-valued standard Brownian motion, \( (\Omega_t)_{t \geq 0} \) is the local boundary time and \( \alpha \) the outward normal.

Let \( \beta \in C^2(\Omega) \), \( \beta > 0 \), and define the additive functional \( (A_t)_{t \geq 0} \) by

\[
A_t = \int_0^t \beta(x) \, dB_s, \quad t \geq 0.
\]

Using the inverse \( (r(t))_{t \geq 0} \) of \( (A_t)_{t \geq 0} \) it is possible to define the time changed process \( (X_t^\beta)_{t \geq 0} \), \( X_0 \in \Omega \), which is a solution of the SDE.

\[
dX_t^\beta = \left( \frac{\nabla \phi(x)}{\left| \nabla \phi(x) \right|} \right) \, dB_t + \beta(x) \, dt, \quad x \in \Omega.
\]

\( (\beta_t)_{t \geq 0} \) is an \( \mathbb{R}^d \)-valued standard Brownian motion. This follows by \( \frac{\partial}{\partial t} \frac{1}{2} \left| \nabla \phi(x) \right|^2 = 0 \) for every \( X_t \in \Omega \), where \( \beta_t \) is called a sticky reflected distorted Brownian motion or distorted Brownian motion with delayed reflection, since the new time scale does not pass the process down if it reaches \( \Gamma \). Moreover, the process spends indeed time on \( \Gamma \) in the sense

\[
\lim_{t \to \infty} \frac{1}{t} \int_0^t 1(x(s) \in \Gamma) \, ds = 0 \quad a.s. \text{ for some fixed constant } c.
\]

The Dirichlet form associated to the distorted Brownian motion with immediate reflection is given by the closure \( \langle E, D(E) \rangle \) of

\[
E(f, g) = \frac{1}{2} \int \nabla f \cdot \nabla g \, d\mu, \quad f, g \in C^2_{\Gamma}(\Omega), \quad \text{on } L^2(\mu, \alpha).
\]

The local time \( (\mathbf{S}_t^\alpha)_{t \geq 0} \) is in Revuz correspondence with the surface measure \( \sigma_{\Omega,\Gamma} \) and therefore, the Revuz measure of \( (A_t)_{t \geq 0} \) is given by \( \alpha \, d\sigma_{\Omega,\Gamma} \). As a consequence, the Dirichlet form \( \langle E, D(E) \rangle \) is associated to the sticky reflected distorted Brownian motion on \( \Omega \) for \( X_{\Omega,t} \in \Omega \) under mild assumptions on the boundary \( \Gamma \), the interaction and the densities \( \alpha \) and \( \beta \). Since the process spends indeed time on \( \Gamma \), we provide additionally an optional diffusion along \( \Gamma \).

This interacting particle system provides a model for the dynamics of molecules in a chromatography tube.

**Theorem 1**

Assume that \( \alpha \) and \( \beta \) are \( \mathcal{C}^2 \)-smooth functions, \( \mu = \mu_0 + \mu_1 \) and there exist \( \varphi, \psi \in \mathcal{C}^2_{\Gamma} \) such that \( \varphi > 0 \) and \( \psi > 0 \).

\[
\langle E, f \rangle = \frac{1}{2} \int \nabla f \cdot \nabla g \, d\mu + \frac{1}{2} \int \nabla f \cdot \nabla \varphi \, d\mu + \frac{1}{2} \int \nabla g \cdot \nabla \psi \, d\mu - \int (\varphi \psi - \psi \varphi) \, d\mu_0
\]

for \( f, g \in C^2_{\Gamma}(\Omega) \). The statement of Theorem 1 holds if \( \varphi \) and \( \psi \) are \( \mathcal{C}^2 \)-smooth functions and \( \mu_0 \) is a measure on \( \mathbb{R}^d \) with finite first moment.

The proof of Theorem 2 is based on the general construction scheme presented in [1] using regularity results for elliptic PDEs with Wentzell boundary conditions.

**Condition C4**

\( \alpha \) is \( \mathcal{C}^2 \)-smooth, \( \alpha > 0 \) and \( \psi \in \mathcal{C}^2_{\Gamma} \) such that \( \psi > 0 \).

\[
\alpha = \frac{1}{2} \left| \nabla \phi(x) \right|^2, \quad x \in \Omega, \quad \text{and } \beta = \frac{1}{2} \left| \nabla \phi(x) \right|^2, \quad x \in \Gamma.
\]

Theorem 2

Suppose that \( \alpha, \beta \) and \( \mu = \mu_0 + \mu_1 \) are as in Theorem 1. Then there exists a conservative diffusion process \( (X_t)_{t \geq 0} \) with state space \( \mathbb{R}^d \) which is properly associated with \( \langle E, D(E) \rangle \) on \( L^2(\mu, \alpha) \) and \( \mu_0 = \mu_{initial} \). The process \( (X_t)_{t \geq 0} \) is the \( \mathcal{C}^2 \)-smooth family of \( \mathcal{C} \)-smooth functions, \( \alpha = \frac{1}{2} \left| \nabla \phi(x) \right|^2, \quad x \in \Omega, \quad \text{and } \beta = \frac{1}{2} \left| \nabla \phi(x) \right|^2, \quad x \in \Gamma. \)

The proof of Theorem 2 is based on the general construction scheme presented in [1] using regularity results for elliptic PDEs with Wentzell boundary conditions.

**Product form of \( N \) pre-Dirichlet forms**

Let \( \alpha_i, \beta_i, i = 1, \ldots, N, \) fulfill C1 and C2. Denote by \( \mathcal{C}^2(\Omega_{2N}) \) the according forms. Define \( \langle \phi_{2N}, \phi_{2N} \rangle_{\mathcal{C}^2(\Omega_{2N})}, \lambda = \lambda_{2N} \) by

\[
\langle \phi_{2N}, \phi_{2N} \rangle_{\mathcal{C}^2(\Omega_{2N})} = \sum_{i=1}^N \frac{1}{2} \left( \int \nabla \phi_{2N} \cdot \nabla \phi_{2N} \, d\mu_{2N} + \int \nabla \phi_{2N} \cdot \nabla \phi_{2N} \, d\mu_{2N} + \int \nabla \phi_{2N} \cdot \nabla \phi_{2N} \, d\mu_{2N} + \int \nabla \phi_{2N} \cdot \nabla \phi_{2N} \, d\mu_{2N} \right)
\]

for \( \phi_{2N} \in \mathcal{C}^2(\Omega_{2N}) \). This bilinear form is closed on \( \mathcal{C}(\Omega_{2N}), \mu = \mu_{2N} \) is its closure in a regular Dirichlet form and yields \( N \) independent sticky reflected distorted Brownian motions. Next we introduce an interaction.

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