DETERMINATION OF GENERALIZED STRESS INTENSITY FACTORS FOR SHARP V-NOTCHED PLATES UNDER TRANSVERSE BENDING

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1. INTRODUCTION

Sharp V-notches in structural components give rise localized stress concentration which decreases the maximum load-bearing capacity of the component, and may generate a crack or lead to early crack initiation. Notches may be regarded as sharp when the radius of curvature of their tips is very small compared with the length of the notch sides. Sharp angular corners of holes with machined notches can also be considered as sharp notches, and cracks can be regarded as particular sharp notches, for which the included angle is zero. If we take a polar coordinate system centered at the tip of a notch angle $\beta$, the stresses near the notch tip can be expressed as

$$\sigma = K_{1,\lambda} r^{\lambda-1} f_1(\theta) + K_{\infty,\gamma} r^{\gamma-1} f_2(\theta),$$

where $\lambda$ and $\gamma$ are the characteristic values depending on the notch angle between two stress-free boundaries, and $K_{1,\lambda}$ and $K_{\infty,\gamma}$ are called as the generalized stress intensity factors of mode I and mode II, respectively. Thus, for given geometry of components and loading conditions, $K_{1,\lambda}$ and $K_{\infty,\gamma}$ completely characterize the stress state in the region near the notch tip, and are a key factor in solving the problems related to the strength evaluation of materials, crack initiation, and the application to the fatigue of notched materials under the cyclic loading. The limiting case of the notch angle $2\beta = 0$ corresponds to a crack, and

$$\lambda - 1 = \gamma - 1 = -1/2, \theta = 0$$

and

$$\sigma = K_1 r^{-1/2} + K_\infty r^{-1/2},$$

where $K_1$ and $K_\infty$ are widely-used stress intensity factors of linear fracture mechanics. Since the pioneer works of Williams, over the past decades, a number of studies have been conducted on the singular stress fields of the V-notched plates by the various methods of solution, such as collocation methods, finite element methods, boundary element methods, and theoretical analyses. However, only a few works concerned with those problems...
have been studied on experimental procedures, such as the method of caustics\textsuperscript{(13), (14)}, the photoelastic technology\textsuperscript{(15)} and recently the coherent gradient method\textsuperscript{(16)}. In the previous papers, the strain gage method of determination of GSIFs at the sharp V-notched plate have been developed, where the V-notched plate under tension\textsuperscript{(17)} and in plane bending\textsuperscript{(18)} were treated.

In this paper, the strain gage method is applied to the determination of GSIFs at the V-notch of the plate under transverse bending. The Kirchhoff\textsuperscript{(19)} plate theory is used. The eigenfunction expansions together with complex functions are employed to derive the singular strain fields in the neighborhood at the tip of the V-notch including the non-singular terms. Second, using these strain fields, a theory of determining the GSIFs is presented under the condition of mixed mode loading. Finally, following the method, experiments on the specimens with three types of notch angle using strain gages were performed for the mode I loading condition. The finite element analysis is also carried out to compare with the experimental results. The both results show good agreement between them and as a result they are influenced on the notch angle.

2. SINGULAR STRAIN FIELDS AROUND THE V-NOTCH

The plate with a sharp V-notch is subjected to a transverse bending load as shown in Fig.1. Let $(r, 0)$ be a polar coordinate system centered at the tip of the V-notch, such that the line $\theta = 0$ axis is bisector of the notch angle $2\beta$, and the angle between the x-axis and the line perpendicular to the straight side of the plate is denoted by $\omega$. The length of line from the notch tip to the side of the plate, which is perpendicular to the plate side as shown in Fig.1, is denoted by $a$.

On the basis of Kirchhoff plate theory\textsuperscript{(19)}, the strain components $e_{rr}$, $e_{\theta\theta}$, $e_{r\theta}$ of the plate free from lateral loads is expressed in terms of two complex potential functions $\varphi(z)$ and $\psi(z)$:\textsuperscript{(20)}

\begin{align*}
e_{rr} + e_{\theta\theta} &= -2\delta \varphi'(z) + \varphi(z) \tag{1} \\
e_{\theta\theta} - e_{rr} + i e_{r\theta} &= 2\delta z \varphi'(z) + \psi(z) e^{i\theta} \tag{2}
\end{align*}

where $z = r e^{i\theta}$, and $\delta$ denotes a distance from the mid-plane in an un-deformed plate to any point including the surface of the plate. From the stress-strain relations, the stress components $\sigma_{rr}$, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ corresponding to eqs. (1) and (2) are derived:

\begin{align*}
\sigma_{rr} + \sigma_{\theta\theta} &= -\frac{2E\delta}{1-\nu} \varphi'(z) + \varphi(z) \tag{3} \\
\sigma_{\theta\theta} - \sigma_{rr} + 2i\sigma_{r\theta} &= \frac{2\delta}{1+\nu} [z \varphi'(z) + \psi(z)e^{i\theta}] \tag{4}
\end{align*}

where $\nu$ is Poisson’s ratio, and $E$ is Young’s modulus of the plate. To obtain singular strain fields to the present problem, the eigenfunction expansion method is used.

We assume the complex potential functions $\varphi(z)$ and $\psi(z)$ near the tip of the V-notched plate analogous to the plane problems\textsuperscript{(7)} as follows:

\begin{align*}
\varphi(z) &= \sum_{n=\pm 1}^{\infty} \left[ A_n z^{1/\alpha} + i B_n z^{-1/\alpha} \right] \tag{5} \\
\psi(z) &= \sum_{n=\pm 1}^{\infty} \left[ -A_n (\kappa \cos 2(\lambda_n - 1)\alpha + 1 + (\lambda_n - 1)\cos 2\alpha) z^{1/\alpha} \\
&\quad + i B_n (\kappa \cos 2(\gamma_n - 1)\alpha - 1 - (\gamma_n - 1)\cos 2\alpha) z^{-1/\alpha} \right] \tag{6}
\end{align*}

where $A_n$ and $B_n$ are real numbers determined by the boundary conditions, and $\kappa = -(3 + \nu)/(1 - \nu)$. The eigenvalue of $\lambda_n$ and $\gamma_n$ correspond to mode I and mode II, respectively, which satisfy the following characteristic equations:

\begin{align*}
\sin 2\alpha(\lambda_n - 1) + \kappa \lambda_n \sin 2\alpha &= 0 \tag{7} \\
\sin 2\alpha(\gamma_n - 1) - \kappa (\gamma_n - 1) \sin 2\alpha &= 0 \tag{8}
\end{align*}

which are derived from the stress free boundary condition along the two notch sides. It is noted that the eigenvalues are influenced by the notch angles as well as Poisson’s ratio $\nu$. Substituting eqs.(5) and (6) into eqs.(1) and (2), we obtain

\begin{align*}
e_{rr} &= \delta \sum_{n=1}^{\infty} \left[ A_n \lambda_n^{1/\alpha - 1} [-1 - \lambda_n] \cos (\lambda_n - 1) \alpha + 1 + (\lambda_n - 1) \cos 2\alpha + 1 \right] \cos (\lambda_n - 1) \alpha \theta + \\
&\quad + [\kappa \cos 2\alpha(\lambda_n - 1) + (\lambda_n - 1) \cos 2\alpha + 1 \cos (\lambda_n - 1) \alpha + 1] e^{i\theta} \tag{9}
\end{align*}
The first non-zero eigenvalues of \( \lambda_1 \) and \( \gamma_1 \) of the above eqs.(7) and (8) are real, where \( 1/2 < \lambda_1 \leq 1 \) and \( 1/2 < \gamma_1 \leq 1 \) for \( \pi/2 < \alpha \leq \pi \), and therefore the strains in eqs.(9),(10) and (11) have singularity at the notch tip and expressed by

\[
\begin{align*}
\varepsilon_{rr} &= \delta_1 \lambda_1 z^{\lambda_1 - 1}[1 - \lambda_1 - 1] \cos(\gamma_1 - 1) \theta + \\
&+ [\cos 2a(\lambda_1 - 1) + (1 - \lambda_1) \cos 2a + 1] \cos(\gamma_1 - 1) \theta + \\
&+ \delta_1 B_n \gamma_n \gamma_n^{\lambda_1 - 1} [1 + \gamma_n] \cos(\gamma_n - 1) \theta + \\
&+ [\cos 2a(\gamma_n - 1) - (\gamma_n - 1) \cos 2a - 1] \sin(\gamma_n + 1) \theta \\
\varepsilon_{\theta \theta} &= \delta_1 \lambda_1 z^{\lambda_1 - 1}[1 - \lambda_1 - 1] \cos(\gamma_1 - 1) \theta + \\
&+ [\cos 2a(\lambda_1 - 1) + (1 - \lambda_1) \cos 2a + 1] \cos(\gamma_1 + 1) \theta + \\
&+ \delta_1 B_n \gamma_n \gamma_n^{\lambda_1 - 1} [1 + \gamma_n] \cos(\gamma_n + 1) \theta + \\
&+ [\cos 2a(\gamma_n - 1) - (\gamma_n - 1) \cos 2a - 1] \cos(\gamma_n + 1) \theta \\
\varepsilon_{00} &= \delta_1 \lambda_1 z^{\lambda_1 - 1}[1 - \lambda_1 - 1] \cos(\gamma_1 - 1) \theta + \\
&+ [\cos 2a(\lambda_1 - 1) + (1 - \lambda_1) \cos 2a + 1] \cos(\gamma_1 + 1) \theta + \\
&+ \delta_1 B_n \gamma_n \gamma_n^{\lambda_1 - 1} [1 + \gamma_n] \cos(\gamma_n + 1) \theta + \\
&+ [\cos 2a(\gamma_n - 1) - (\gamma_n - 1) \cos 2a - 1] \cos(\gamma_n + 1) \theta
\end{align*}
\]

Similarly, the singular stress components \( \sigma_{rr}, \sigma_{\theta \theta}, \sigma_{00} \) are derived by eqs.(3) and (4) for \( n = 1 \), as follows:

\[
\begin{align*}
\sigma_{rr} &= \sigma(1 - \nu^2)(\lambda_1 \lambda_1^{\lambda_1 - 1}[1 - \lambda_1 - 1] z \cos(\lambda_1 - 1) \theta + \\
&+ [\cos 2a(\lambda_1 - 1) + (1 - \lambda_1) \cos 2a + 1] \cos(\lambda_1 + 1) \theta + \\
&+ \delta_1 B_n \gamma_n \gamma_n^{\lambda_1 - 1} [1 + \gamma_n] \cos(\gamma_n + 1) \theta + \\
&+ [\cos 2a(\gamma_n - 1) - (\gamma_n - 1) \cos 2a + 1] \cos(\gamma_n + 1) \theta \\
\sigma_{\theta \theta} &= \sigma(1 - \nu^2)(\lambda_1 \lambda_1^{\lambda_1 - 1}[1 - \lambda_1 - 1] z \cos(\lambda_1 + 1) \theta + \\
&+ [\cos 2a(\lambda_1 - 1) + (1 - \lambda_1) \cos 2a + 1] \cos(\lambda_1 + 1) \theta + \\
&+ \delta_1 B_n \gamma_n \gamma_n^{\lambda_1 - 1} [1 + \gamma_n] \cos(\gamma_n + 1) \theta + \\
&+ [\cos 2a(\gamma_n - 1) - (\gamma_n - 1) \cos 2a + 1] \cos(\gamma_n + 1) \theta \\
\sigma_{00} &= \sigma(1 - \nu^2)(\lambda_1 \lambda_1^{\lambda_1 - 1}[1 - \lambda_1 - 1] z \cos(\lambda_1 + 1) \theta + \\
&+ [\cos 2a(\lambda_1 - 1) + (1 - \lambda_1) \cos 2a + 1] \cos(\lambda_1 + 1) \theta + \\
&+ \delta_1 B_n \gamma_n \gamma_n^{\lambda_1 - 1} [1 + \gamma_n] \cos(\gamma_n + 1) \theta + \\
&+ [\cos 2a(\gamma_n - 1) - (\gamma_n - 1) \cos 2a + 1] \cos(\gamma_n + 1) \theta
\end{align*}
\]

In this study, the GSIFs of \( K_{I,\lambda} \) and \( K_{II,\gamma} \) are defined, following by Chen et al., as follows:

\[
K_{I,\lambda} = \lim_{r \to 0, \theta = 0} \sqrt{2\pi r^{1-\lambda_1}} \sigma_{rr}(r, \theta)
\]

\[
K_{II,\gamma} = \lim_{r \to 0, \theta = 0} \sqrt{2\pi r^{1-\gamma_1}} \sigma_{rr}(r, \theta)
\]

The coefficient \( (3 + \nu)/(1 + \nu) \) in the right hand side of eq.(19) is added to satisfy the consistency to the plane problems. Substituting eqs.(16) and (17) into eqs.(18) and (19), we obtain

\[
A_1 = K_{I,\lambda} \frac{1 - \nu^2}{2\pi} \frac{1}{\sqrt{E_0 I_{C_I}}} \quad B_1 = K_{II,\gamma} \frac{1 + \nu^2}{2\pi} \frac{1}{\sqrt{E_0 I_{C_{II}}}}
\]

\[
C_1 = \left[ -3 + \nu + \lambda_1 (1 - \nu)^2 \right] - (1 - \nu) [\cos 2\alpha(\lambda_1 - 1) + (\lambda_1 - 1) \cos 2\alpha]
\]

Thus we obtain the singular strain fields near the V-notch in the final form including the non-singular terms \( f_0, f_0, f_0 \) as follows:

\[
\begin{align*}
e_\nu &= -K_{I,\lambda} \frac{(1 - \nu^2)}{2\pi r^{1-\lambda_1}} \left[ C_1 \cos(\lambda_1 + 1) \theta + (\lambda_1 - 1) \cos(\lambda_1 - 1) \theta \right] + \\
&+ \frac{K_{II,\gamma}}{2\pi r^{1-\gamma_1}} \left[ C_{\gamma_1} \sin(\gamma_1 + 1) \theta + (\gamma_1 - 1) \sin(\gamma_1 - 1) \theta \right] + f_0
\end{align*}
\]

\[
\begin{align*}
e_\nu &= -K_{I,\lambda} \frac{(1 - \nu^2)}{2\pi r^{1-\lambda_1}} \left[ C_1 \cos(\lambda_1 + 1) \theta + (\lambda_1 - 1) \cos(\lambda_1 - 1) \theta \right] - \\
&- \frac{K_{II,\gamma}}{2\pi r^{1-\gamma_1}} \left[ C_{\gamma_1} \sin(\gamma_1 + 1) \theta + (\gamma_1 - 1) \sin(\gamma_1 - 1) \theta \right] + f_0
\end{align*}
\]

\[
\begin{align*}
e_\nu &= -K_{I,\lambda} \frac{(1 - \nu^2)}{2\pi r^{1-\lambda_1}} \left[ C_1 \cos(\lambda_1 + 1) \theta + (\lambda_1 - 1) \cos(\lambda_1 - 1) \theta \right] + \\
&+ \frac{K_{II,\gamma}}{2\pi r^{1-\gamma_1}} \left[ C_{\gamma_1} \sin(\gamma_1 + 1) \theta + (\gamma_1 - 1) \sin(\gamma_1 - 1) \theta \right] + f_0
\end{align*}
\]

Similarly, for the singular stress fields:

\[
\begin{align*}
\sigma_{rr} &= \frac{-K_{I,\lambda}}{\sqrt{2\pi r^{1-\lambda_1}}} \times \\
&\times \left[ C_1 (1 - \nu^2) \cos(\lambda_1 + 1) \theta + (1 + 3\nu + \lambda_1 (1 - \nu^2) \cos(\lambda_1 - 1) \theta \right] + \\
&+ \frac{K_{II,\gamma}}{\sqrt{2\pi r^{1-\gamma_1}}} \times \\
&\times \left[ C_{\gamma_1} (1 - \nu^2) \sin(\gamma_1 + 1) \theta + (1 + 3\nu + \gamma_1 (1 - \nu^2) \sin(\gamma_1 - 1) \theta \right] + f_3 \times \\
\end{align*}
\]
where $\varepsilon_{i3}^*, \varepsilon_{i03}^*, \varepsilon_{o3}^*$ is also non-singular terms. If we put $\alpha = \pi$, $\lambda_1 = \gamma_1 = 1/2$, i.e., for the special case of a crack, the singular strain and stress fields are obtained from eqs.(24)–(29), as follows:

$$
\varepsilon_{ii} \equiv \frac{K_{i3}}{\sqrt{2\pi(3 + \nu)}} \left( \frac{(7 + \nu)(\nu - 1)}{2} \right) + f_{i3}^i + f_{i03}^i + f_{oi3}^i 
$$

$$
\sigma_{ii} \equiv \frac{K_{i3}}{\sqrt{2\pi(3 + \nu)}} \left( \frac{(7 + \nu)(\nu - 1)}{2} \right) + f_{i3}^i + f_{i03}^i + f_{oi3}^i 
$$

These equations are coinciding with those obtained by Williams\textsuperscript{21} for the crack problem under transverse bending.

### 3. PROCEDURE OF DETERMINATION OF GSIFs

In the mixed loading condition, to separate the GSIFs $K_{i3}$ and $K_{i1}$, we must measure the strains along more than two points in any directions and the line extending from the notch tip. Basically, although both strains $\varepsilon_{ir}$ and $\varepsilon_{o0}$ can be available to obtain $K_{i3}$ and $K_{i1}$, we used $\varepsilon_{ir}$ in this study. The strip strain gages consisting of five measuring grids are positioned along the two directions of $\theta = \theta_1$, and $\theta_2$ measured from the extension of the bisector of the notch angle as shown in Fig.2. We denote the strains of the five points in the directions $\theta_1$ and $\theta_2$ as $\varepsilon_{i1r}$ and $\varepsilon_{i2r}$, respectively. Moreover, the distances from the notch tip to each center of strain gage position in the direction of $\theta_1$ and $\theta_2$ are denoted by $r_{i1}$ and $r_{i2}$, respectively (see Fig.2). Substituting the measured quantities of $\varepsilon_{i1r}$ and $\varepsilon_{i2r}$ into eq.(24), we obtain the basic expressions to separate GSIFs of $K_{i3}$ and $K_{i1}$ under mixed mode conditions as follows:

![Fig.2 Strain gage positions and directions](image)
\[ Z_i(e_{ij}, r_{ji}) = A_i X_i(r_{ji}) + B_i Y_i(r_{ji}) + K_{1_i} \]  
\[ Z_2(e_{ij}, r_{ji}) = A_2 X_2(r_{ji}) + B_2 Y_2(r_{ji}) + K_{2_i} \]  
\[ (i = 1, 2, \ldots, 5) \]  
(36)  
(37)

where \( X_j \) and \( Y_j \) are the functions of only \( r_{ji} \), and \( A_j \) and \( B_j \) are constants that do not include \( r_{ij} \) and \( e_{rji} \) (see Appendix). Equations (36) and (37) express the plane in the three dimensional coordinates \((X_j, Y_j, Z_j)\) and show that \( K_{1_k} \) and \( K_{1_y} \) can be obtained from the intersection of the \( Z_j \) axis (see Fig.3) and the plane constructed by the least squares method using the strains \( e_{rji} \) measured in the two directions \( \theta_1 \) and \( \theta_2 \) as shown in Fig.2.

4. EXPERIMENTAL PROCEDURE AND FINITE ELEMENT ANALYSIS

4.1 Test Specimen and Experiment

Experiments are performed to demonstrate the method described above. Transverse bending specimens with \( w=500 \)mm width and \( h=100 \)mm length made of aluminum 5052 alloy, which fabricated from a 8mm thick plate, were used (see Fig.1). Three types of notch angles corresponding to the mode I loading condition were machined by the wire electric discharge machine with a wire of 0.03mm diameter (see Table 1). Thus, the radius of curvature of the notch tip is 0.015 mm and is very small compared with the notch sides, and it may be regarded as the sharp notch. Mechanical properties of the specimens are summarized in Table 2. Strain gages with five element grids with gage length 1mm (Kyowa Company, KFG-1-120-Da-23N-10C2) were positioned along the line \( \theta_1 = \theta_2 = 90 \)deg to obtain still larger strains. The CCD camera with 0.01mm accuracy was used to measuring the distances from the each strain gage position \( r_{ji} \) \( (j = 1, 2, \ldots, 5) \) (see Table 3). The suitable location of strain gage was referred to the results studied by Dally and Sanford22) for the crack problem. The transverse bending moment was carried out by the four-point bending apparatus from 100N to 220N by the digital-testing machine. The magnitude of the loads are determined to hold the small scale yielding condition in linear fracture mechanics, specifically we took the radius of plastic zone size as within 0.1mm. Here, the following non-dimensional notations are used:

\[ K_{1_i}^* = \frac{K_{1_i}}{\sigma_y \sqrt{\pi a_{1_i}}} , \quad K_{2_i}^* = \frac{K_{2_i}}{\sigma_y \sqrt{\pi a_{2_i}}} \]  
(38),(39)

where \( \sigma_y = 3P/ \ell^2 \) and \( \ell \) is the distance from the support to the loading position of the plate (see Fig.1). It should be noted that the theoretical values \( K_{1_i}^* \) and \( K_{2_i}^* \) may be independent of the load for the given notch angles and shapes.

4.2 Finite Element Analysis

Finite element analysis (FEA) based on the Kirchhoff plate theory was performed to compare with the results of the experiments. Configurations and boundary conditions of the specimens used in the analysis are the same as those in the experiments. In the finite element analysis, we used the [ANSYS] for each specimen, following by the method outlined in the previous chapter. Figure 4 shows the typical finite element mesh for the analysis, consisting 8680 elements and 8109 nodes, and the most fine mesh length is \( 10^{-7} \)m near the

### Table 1 Specimen dimensions

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( 2\beta ) (deg)</th>
<th>( \lambda_i )</th>
<th>( \gamma_i )</th>
<th>( \omega ) (deg)</th>
<th>( h ) (mm)</th>
<th>( w ) (mm)</th>
<th>( a ) (mm)</th>
<th>( t ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td>100</td>
<td>500</td>
<td>25</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>0.55498</td>
<td>0.53625</td>
<td>0</td>
<td>100</td>
<td>500</td>
<td>25</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>60</td>
<td>0.62031</td>
<td>0.58097</td>
<td></td>
<td>100</td>
<td>500</td>
<td>25</td>
<td>8</td>
</tr>
</tbody>
</table>

### Table 2 Mechanical properties of the specimen

<table>
<thead>
<tr>
<th>Young's Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Yield Strength (MPa)</th>
<th>Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.3</td>
<td>0.34</td>
<td>91.1</td>
<td>197</td>
</tr>
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</table>
Table 3 Distances from the notch tip to the strain gage positions

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$r_{i1}$ (mm), $\theta_1 = 90$ deg. ($i=1,2,\ldots,5$)</th>
<th>$r_{21}$ (mm), $\theta_2 = -90$ deg. ($i=1,2,\ldots,5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.78 3.78 5.78 7.78 9.78 1.65 3.65 5.65 7.65 9.65</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.73 3.73 5.73 7.73 9.73 1.61 3.61 5.61 7.61 9.61</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>2.24 4.24 6.24 8.24 10.24 1.87 3.87 5.87 7.87 9.87</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Strains $\varepsilon_{r_{i1}}$ and $\varepsilon_{r_{21}}$ ($i=1,2,\ldots,5$) in the direction of $\theta_1 = 90$ deg. and $\theta_2 = -90$ deg.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Load (N)</th>
<th>$\varepsilon_{r_{i1}} \times 10^{-6}$, $\theta_1 = 90$ deg. ($i=1,2,\ldots,5$)</th>
<th>$\varepsilon_{r_{21}} \times 10^{-6}$, $\theta_2 = -90$ deg. ($i=1,2,\ldots,5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
<td>71 53 43 39 36 68 49 40 36 31</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>130</td>
<td>66 53 48 44 68 65 52 47 41</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>160</td>
<td>81 58 53 105 76 63 55 49</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>190</td>
<td>95 71 66 132 95 80 68 62</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>220</td>
<td>114 85 78 154 113 93 81 74</td>
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</tr>
<tr>
<td>C</td>
<td>100</td>
<td>53 33 33 93 67 57 52 46</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>130</td>
<td>68 49 44 93 67 57 52 46</td>
<td></td>
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<tr>
<td>B</td>
<td>160</td>
<td>86 70 63 118 84 72 66 57</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>190</td>
<td>104 85 75 140 102 84 77 67</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>220</td>
<td>117 83 74 156 116 98 89 75</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>100</td>
<td>57 43 40 70 54 48 41 40</td>
<td></td>
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<tr>
<td>C</td>
<td>130</td>
<td>73 58 56 93 72 65 56 51</td>
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</tr>
<tr>
<td>A</td>
<td>160</td>
<td>88 76 69 114 90 81 69 63</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>190</td>
<td>102 80 76 132 102 90 79 71</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>220</td>
<td>124 106 97 156 122 109 95 87</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4 An example of finite element mesh for $2\beta =30\text{deg.}$ and vertical lengths of $y$ enclosing by squares.
tip of the V-notch. Three dimensional representation of the strain fields \(\varepsilon_{rr}\) near the sharp V-notched plate under \(P=220\text{N}\) for notch angle \(2\beta = 30\text{deg}\) is shown in Fig. 5. This figure shows clearly that the strain field is considerably affected by the angle \(\theta\) around the V-notch. The strains from the tips of the notch to the each collocation point \(r_{ji}\) and the angle \(\theta_1 = -\theta_2 = 90\text{deg}\) as shown in Fig.2 were employed in the same way as those in the case of the experiment.

5. RESULTS AND DISCUSSION

For the specimens with the three types of notch angle, the measured strains \(\varepsilon_{rr}\) \((j=1,2, i=1,2, \cdots, 5)\) by strain gages in the two directions of \(\theta_1 = -\theta_2 = 90\text{deg}\) are shown in Table 4. The strains obtained by experiments are introduced into eqs.(36) and (37), and experimental values of \(K_{1,\lambda_i}\) are determined by the procedure stated above. These experimental values are compared with the finite element results.

Figure 6 shows the relation between the non-dimensional stress intensity factor \(K^*_{1,\lambda_i}\) and the bending stresses for the fixed values of \(2\beta = 0\text{deg}\). In Fig.6, the closed form solution for an infinite plate obtained by Sih\(^{29}\) et.al is also indicated for the comparison with experimental and the FEA results. In this case, the effects of stress free surface of the edge and Poisson’s ratio of semi-infinite plate are almost negligible\(^{24}\). It should be noted again that the \(K^*_{1,\lambda_i}\) is independent of the given bending stresses for the non-dimensional quantity. The results of the three cases in Fig.6 show good agreement between them, particularly the FEA result agrees well with the closed form

\[
\begin{align*}
\text{Fig.5} & \quad \text{Three dimensional representation of the strain field near the V-notch} \\
\text{Fig.6} & \quad \text{Comparison of the experimental and FEA results of } K^*_{1,\lambda_i} \text{ for various bending stresses in the case of } 2\beta = 0\text{deg.} \\
\text{Fig.7} & \quad \text{Comparison of the experimental and FEA results of } K^*_{1,\lambda_i} \text{ for various bending stresses in the case of } 2\beta = 30\text{deg.} \\
\text{Fig.8} & \quad \text{Comparison of the experimental and FEA results of } K^*_{1,\lambda_i} \text{ for various bending stresses in the case of } 2\beta = 60\text{deg.}
\end{align*}
\]
solution. Therefore, the FEA result may be available for an indicator of accuracy of experimental results. For $2\beta = 30^\circ$, a comparison of the experimental and FEA results of $K_{I,\alpha}^{\alpha}$ is shown in Fig.7. The maximum value of difference between the experimental and FEA is within 10\%.

Fig.8 shows the case of $2\beta = 60^\circ$. It can be seen from the figure that the maximum difference between the experimental and FEA results is larger than that of Fig.7.

6. CONCLUSION

The method of determination of the generalized stress intensity factors was developed to the bending problem on the basis of the Kirchhoff plate theory. By measuring the strains on the two lines extending from the bisector of the notch angle, we can separate the mixed mode condition into the independent generalized stress intensity factors. Experiments on the specimens with three types of notch angles $2\beta = 0^\circ$, $30^\circ$ and $60^\circ$ for the mode I loading conditions, were performed by using strain gages, and are compared with the finite element analysis. The both results show good agreement between them.

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Appendix

\[ Z_1(\varepsilon_{ij}, r_j) = \sqrt{2\pi} \frac{\varepsilon_{i1} r_1^{1-\gamma_1} + \varepsilon_{i2} r_2^{1-\gamma_1}}{f_{i1}(\alpha_1, \alpha, \theta_1) \cdot (r_1^{1-\gamma_1} + r_2^{1-\gamma_1})} \]  
\[ Z_2(\varepsilon_{ij}, r_j) = \sqrt{2\pi} \frac{\varepsilon_{i1} r_1^{1-\lambda_1} - \varepsilon_{i2} r_2^{1-\lambda_1}}{f_{i2}(\gamma_1, \alpha, \theta_2) \cdot (r_1^{1-\lambda_1} + r_2^{1-\lambda_1})} \]  
\[ A_1 = \frac{\sqrt{2\pi}}{f_{i1}(\lambda_1, \alpha, \theta_1)} \cdot A_2 = \frac{\sqrt{2\pi}}{f_{i2}(\lambda_1, \alpha, \theta_2)} \]

\[ B_1 = \sqrt{2\pi} \frac{f_{i3}(\omega, \theta_2)}{f_{i1}(\alpha_1, \theta_1)}, \quad B_2 = \frac{f_{i3}(\omega, \theta_2)}{f_{i2}(\alpha_1, \theta_2)} \]  
\[ X_1 = \frac{r_1^{1-\gamma_1}}{r_1^{1-\gamma_1} + r_2^{1-\gamma_1}}, \quad X_2 = \frac{r_1^{1-\lambda_1}}{r_1^{1-\lambda_1} + r_2^{1-\lambda_1}} \]  
\[ Y_1 = \frac{r_1^{1-\gamma_1}}{r_1^{1-\gamma_1} + r_2^{1-\gamma_1}}, \quad Y_2 = \frac{r_1^{1-\lambda_1}}{r_1^{1-\lambda_1} + r_2^{1-\lambda_1}} \]

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