Notations

- (ω_t, P) : 0 を出発点とする $\mathbb{R}^d$ 上の Lévy 過程.
- $P[e^{-i\xi\cdot\omega_t}] = e^{-tp(\xi)}$: ($(\omega_t, P)$ の symbol)
  $P[\cdot]$: P に関する期待値.
- $V_t(\omega) := \{(s, x) : s \in (0, t], x \in U(\omega_s)\}$.
  ($U(x)$: $|U(x)| = 1$ となる閉球)

- $\mathcal{M} := \{\eta : \mathbb{R}_+ \times \mathbb{R}^d$上の整数値測度,
  $A \in \mathcal{B}(\mathbb{R}_+ \times \mathbb{R}^d), |A| < \infty \Rightarrow \eta(A) < \infty\}$
- $\mathcal{G} := \sigma[\eta(A) : A \in \mathcal{B}(\mathbb{R}_+ \times \mathbb{R}^d)]$
- $\mathcal{G}_t := \sigma[\eta_t(A) : A \in \mathcal{B}(\mathbb{R}_+ \times \mathbb{R}^d)]$
  $(\eta_t(A) := \eta(A \cap ((0, t] \times \mathbb{R}^d)))$
- Q : $(\mathcal{M}, \mathcal{G})$ 上の Poisson ランダム測度 :
  $\forall A_1, \ldots, A_n \in \mathcal{B}(\mathbb{R}_+ \times \mathbb{R}^d)$
  $A_i \cap A_j = \emptyset$, $|A_i| < \infty$,

$$Q\left(\bigcap_{i=1}^{n}\{\eta(A_i) = k_i\}\right) = \prod_{i=1}^{n} \exp(-|A_{i|})\frac{|A_i|^{k_i}}{k_i!}$$

Strong and Weak disorder

$\beta \in \mathbb{R}$: パラメータ.

- $Z_t := P[\exp(\beta\eta(V_t))]$ (分配函数)
- $Q[Z_t] = \exp((e^\beta - 1)t)$.
- $\lambda := e^\beta - 1$.
- $W_t := e^{-\lambda t} Z_t$ (正規化された分配函数)

$$Q(W_\infty = 0) = 1 \quad (\textbf{Strong disorder})$$
$$Q(W_\infty > 0) = 1 \quad (\textbf{Weak disorder})$$

($\because$ W_t は Q-マルチンゲール.
$\Rightarrow \exists W_\infty := \lim_{t\to\infty} W_t$ Q-a.e.
$\Rightarrow$ Kolmogorov's 0-1 law より)

Main Theorem

(i) $d = 1,\ 1 < \exists\alpha \leq 2,$
$\limsup_{\xi \to 0} p(\xi)\|\xi\|^{-\alpha} < \infty$
$\Rightarrow$
$\forall\beta \neq 0,\ Q(W_\infty = 0) = 1.$

(ii) (ω_t, P) が transient
$\Rightarrow$
$-\infty \leq \exists\beta_0 < 0 < \exists\beta_1 < \infty,$
$\forall\beta \in (\beta_0, \beta_1),\ Q(W_\infty > 0) = 1.$

Examples

- α-stable process:

$p(\xi) = \|\xi\|^{\alpha}$ $(0 < \alpha < 2)$.

  $d = 1, 1 < \alpha < 2 \Rightarrow$ (i) の条件を満たす.

  $d > \alpha \Rightarrow$ (ii) の条件を満たす.

- relativistic α-stable process:

$p(\xi) = (\|\xi\|^2 + m^{2/\alpha})^{\alpha/2} - m$

$(0 < \alpha < 2, m > 0)$.

  $d = 1 \Rightarrow$ (i) の条件を満たす.

  $d \geq 3 \Rightarrow$ (ii) の条件を満たす.

- geometric α-stable process:

$p(\xi) = \log(1 + \|\xi\|^{\alpha})$ $(0 < \alpha \leq 2)$.

  $d = 1, 1 < \alpha \leq 2 \Rightarrow$ (i) の条件を満たす.

  $d > \alpha \Rightarrow$ (ii) の条件を満たす.