German-Japanese conference on

Stochastic Analysis and Applications

Date: 31 August, 2015 (Mon.) – 4 September, 2015 (Fri.)

Venue: Sakura Hall (Katahira Campus, Tohoku University)
Sendai, Miyagi, Japan

Program

31 August (Mon.)

9:00–10:15	Registration
10:15–10:50	Michael Röckner (University of Bielefeld) Infinite dimensional continuity and Fokker-Planck-Kolmogorov equations
11:00–11:35	Ichiro Shigekawa (Kyoto University) The logarithmic Sobolev inequality and the convergence of a semigroup in the Zygmund space
11:45–12:20	Panki Kim (Seoul National University) Minimal thinness for jump processes
12:20–13:40	Lunch break
13:40–14:15	Jun Masamune (Tohoku University) Variational convergence on Riemannian manifolds
14:25–15:00	Ryokichi Tanaka (Tohoku University) Random walks on hyperbolic groups: entropy and speed
15:10–15:45	Xiangdong Li (Chinese Academy of Science) W-entropy formulas and rigidity theorems on Wasserstein space over Riemannian manifolds
16:05–16:40	Kazumasa Kuwada (Tokyo Institute of Technology) A dimensional Wasserstein contraction characterizing the curvature-dimension condition
16:50–18:30	Poster session

18:30– Banquet

1 September (Tue.)

10:15–10:50 Kazuhiro Kuwae (Fukuoka University)
Analytic characterization of gaugeability for generalized Feynman-Kac functionals and its applications

11:00–11:35 Nikola Sandrić (Technical University of Dresden and University of Zagreb)
Long-time behavior of Lévy-type processes: transience, recurrence and ergodicity

11:45–12:20 Masaki Wada (Tohoku University)
Large time asymptotics of Feynman-Kac functionals for symmetric α-stable processes

12:20–13:40 Lunch break

13:40–14:15 Yuichi Shiozawa (Okayama University)
Escape rate of symmetric Markov processes

14:25–15:00 Walter Hoh (University of Bielefeld)
Markov processes with jumps and nonlocal generators

15:10–15:45 Hideki Tanemura (Chiba University)
Systems of infinitely many Brownian motions with long ranged interaction

16:05–16:40 Martin Grothaus (Technical Univesity of Kaiserslautern)
On the stochastic heat equation with sticky reflected boundary condition

16:50–17:25 Song Liang (University of Tsukuba)
Stochastic Newton equation with absorbing area

2 September (Wed.)

9:30–10:05 Naotaka Kajino (Kobe University)
Localized upper bounds of heat kernels for diffusions via a multiple Dynkin-Hunt formula

10:15–10:50 Sebastian Andres (University of Bonn)
Heat kernel estimates for random walks with degenerate weights

11:00–11:35 Makoto Nakashima (University of Tsukuba)
Phase transitions of random walk pinning model

11:45–12:20 Stefan Neukamm (Technical University of Dresden)
A regularity theory for elliptic systems with random coefficients

12:20– Lunch break
Excursion

18:30– Conference dinner

3 September (Thu.)

10:15–10:50 Free discussion

11:00–11:35 Fumio Hiroshima (Kyushu University)
Quantum field theory by Gibbs measures on cadlag path space

11:45–12:20 Martin Huesmann (University of Bonn)
The geometry of multi marginal Skorokhod embedding

12:20–13:40 Lunch break

13:40–14:15 Xue-Mei Li (University of Warwick)
Stochastic homogenization in geometry

14:25–15:00 Yukio Nagahata (Niigata University)
Spectral gap for surface diffusion

15:10–15:45 Gerald Trutnau (Seoul National University)
Recurrence criteria for diffusion processes generated by divergence free perturbations of non-symmetric energy forms

16:05–16:40 Hiroshi Kawabi (Okayama University)
Functional CLTs for non-symmetric random walks on crystal lattices

16:50–17:25 Wolfgang Löhr (University of Duisburg-Essen)
Invariance principle for variable speed random walks on trees

4 September (Fri.)

9:30–10:05 Max von Renesse (University of Leipzig)
Modified Arratia flow and Wasserstein diffusion

10:15–10:50 Nobuaki Naganuma (Tohoku University)
Error analysis for approximations to one-dimensional SDEs via perturbation method

11:00–11:35 Yuzuru Inahama (Nagoya University)
Large deviations for rough path lifts of Donsker-Watanabe's delta functions

11:45–12:20 Martin Hairer (University of Warwick)
Modelling a random rubber band

12:20–13:40 Lunch break

13:40–14:15 Seiichiro Kusuoka (Tohoku University)
Continuity and bounds of the density functions of the solutions to path-dependent stochastic differential equations

14:25–15:00 Karl-Theodor Sturm (University of Bonn)
Optimal transport, Brownian motion, and super-Ricci flow for metric measure spaces

Organizers: Shigeki Aida (Tohoku University)
Hiroshi Kawabi (Okayama University)
Seiichiro Kusuoka (Tohoku University)
Kazumasa Kuwada (Tokyo Institute of Technology)
Yuichi Shiozawa (Okayama University)
Masayoshi Takeda (Tohoku University)

Local organizers: Shigeki Aida (Tohoku University)
Yuu Hariya (Tohoku University)
Seiichiro Kusuoka (Tohoku University)
Masayoshi Takeda (Tohoku University)

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- Japan Society for the Promotion of Science (JSPS Kakenhi),
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- Tohoku Forum for Creativity

Poster Session

31 August (Mon.) 16:50–18:30

Posters will have been exhibited through the whole conference

Yoshihiro Abe (Kyoto Univ.)	Maximum and minimum of local times for two-dimensional random walk
Takafumi Amaba (Ritsumeikan Univ.)	Convergence implications via dual flow method
Syota Esaki (Tokyo Inst. Tech.)	Infinite particle systems of long range jumps with long range interactions
Torben Fattler (TU Kaiserslautern)	Disease spreading models within the framework of two-component configuration spaces in continuum
Julian Hollender (TU Dresden)	Unbounded viscosity solutions of non-dominated HJB-equations
Masato Hoshino (Univ. of Tokyo)	KPZ equation with fractional derivatives of white noise
Yu Ito (Osaka Univ.)	Integration of controlled rough paths via fractional calculus
Yosuke Kawamoto (Kyushu Univ.)	Finite particle approximation of interacting Brownian motion
Kyung-youn Kim (Seoul Nat. Univ.)	Heat kernel estimates for symmetric Markov processes in $C^{1,\rho}$ open sets and its application
Eva Kopfer (Univ. of Bonn)	Minimizing movement scheme for time dependent metrics on probability space
Franziska Kühn (TU Dresden)	Existence and estimates of moments for Lévy-type processes
Katharina von der Lühe (Univ. of Bielefeld)	Pathwise uniqueness for SDEs with non-regular drift and non-constant diffusion
Yusuke Miura (Tohoku Univ.)	The conservativeness of Girsanov transformed symmetric Markov processes
Chikara Nakamura (Kyoto Univ.)	Lamplighter random walks on fractals

Izumi Okada (Tokyo Inst. Tech.)	Geometry structures of favorite sites of random walk range
Kohei Suzuki (Kyoto Univ.)	Convergence of Brownian motions on $\mathrm{RCD}^*(K, N)$ spaces
Robert Voßhall (TU Kaiserslautern)	Interacting particle systems with sticky boundary
Melchior Wirth (FSU Jena)	Diffusion determines the recurrent graph
Lu Xu (Univ. of Tokyo)	Central limit theorem for stochastic heat equations in random environments

Abstracts

Heat kernel estimates for random walks with degenerate weights

Sebastian Andres (University of Bonn)

In this talk we present Gaussian-type upper bounds on the heat kernel for continuous-time random walks on a graph with unbounded weights under an ergodicity assumption. The proof is based Davies' perturbation method, where one key result is a maximal inequality for the perturbed heat kernel obtained by a Moser iteration. This is joint work with Jean-Dominique Deuschel and Martin Slowik.

On the stochastic heat equation with sticky reflected boundary condition

Martin Grothaus (Technical Univesity of Kaiserslautern)

In this talk we study the stochastic heat equation with sticky reflected boundary condition. Dirichlet form techniques are used to construct and characterize its solution. The obtained process already for some time is conjectured to be the scaling limit of the dynamical wetting model, also known as Ginzburg–Landau dynamics with pinning and reflection competing on the boundary. For the second part of this talk it is planned to discuss the progress on this problem.

Modelling a random rubber band

Martin Hairer (University of Warwick)

A rubber band constrained to remain on a manifold evolves by trying to shorten its length, eventually settling on some minimal closed geodesic, or collapsing entirely. It is natural to try to consider a noisy version of such a model where each segment of the band gets pulled in random directions. Trying to build such a model turns out to be surprisingly difficult and generates a number of nice geometric insights, as well as some beautiful algebraic and analytical objects.

Quantum field theory by Gibbs measures on cadlag path space

Fumio Hiroshima (Kyushu University)

We consider a semi-relativistic Pauli-Fierz model (SRPF) in QED. This describes an interaction system governed by a relativistic Schrödinger operator $\sqrt{-\Delta+m^2}+V$ minimally coupled to a massless quantized radiation field. The Hamiltonian of SRPF is given as a self-adjoint operator H on the tensor product Hilbert space, $\mathcal{H}=L^2(\mathbb{R}^d)\otimes Fock$, by

$$H=\sqrt{(-i\nabla_x\otimes 1+\sqrt{\alpha}A(x))^2+m^2}+V\otimes 1+1\otimes H_{\rm rad}.$$

Here $A_\mu(x)$ denotes a Gaussian random process and $H_{\rm rad}$ the 2nd quantization of $\sqrt{-\Delta}$. In this talk we show (1) H has a unique ground state $\phi_g\in\mathcal{H}$, (2) there exists a Gibbs measure μ_∞ associated with ϕ_g on a discontinuous path space D, (3) for some observable F, ground state expectation $(\phi_g,F\phi_g)=\int_D F_g\mu_\infty$ is given, (4) several properties (spatial decay, Gaussian domination, density of bosons) of ϕ_g is derived from representation in (3).

References

[1] T. Hidaka and F. Hiroshima, Self-adjointness of semi-relativistic Pauli-Fierz models, 2014 preprint.

[2] F. Hiroshima, Functional integral approach to semi-relativistic Pauli-Fierz models, *Adv. in Math.* **259** (2014), 784-840.

[3] M. Hirokawa, F. Hiroshima and J. Lorinczi, Spin-boson model through a Poisson-driven stochastic process, *Math. Z.* **277** (2014),1165-1198.

[4] F. Hiroshima and J. Lorinczi, Functional integral representations of the Pauli-Fierz model with spin 1/2, *J. Funct. Anal.* **254** (2008), 2127-2185.

Markov processes with jumps and nonlocal generators

Walter Hoh (University of Bielefeld)

The generator of a Markov process with jumps is a nonlocal operator. In Eulidian space in many cases these generators can be represented as pseudo differential operators. The symbols of these operators have the property that they are negative definite functions with respect to the co-variable. In the talk we discuss how specfific properties of these symbols can be used to derive the existence and properties of corresponding semigroups.

The geometry of multi marginal Skorokhod embedding

Martin Huesmann (University of Bonn)

During the last 50 years the Skorokhod embedding problem has become an important classical problem in probability theory and a number of solutions with particular optimality properties have been constructed. Recently a unified derivation of many of these solutions has been obtained through a new approach inspired by the theory of optimal transport.
Using the original techniques from stochastic analysis, the multi-period version of the Skorokhod problem seems difficult and only limited results are available: Henry-Labordere, Obloj, Spoida, and Touzi derive the multi-marginal Azema-Yor embedding under additional technical conditions and recently the multi-marginal Root embedding has been obtained by Cox, Obloj, and Touzi.
Here we show that the transport approach can also be used to extend the classical optimal solutions to the multi-marginal Skorokhod problem. In particular we establish that these constructions share a common geometric structure. This has further applications to the martingale optimal transport problem.
(joint work with M. Beiglböck and A. Cox)

Large deviations for rough path lifts of Donsker-Watanabe's delta functions

Yuzuru Inahama (Nagoya University)

In 1993 Takanobu and Watanabe presented a large deviation principle (LDP) of Freidlin-Wentzell type for solutions of stochastic differential equations (SDEs) under the strong Hörmander condition anywhere. Unlike in the usual LDP of this type, the probability measures are not the push-forwards of the (scaled) Wiener measure, but the push-forwards of the measures of finite energy which is defined by the composition of the solutions of SDEs and the delta functions (i.e., Watanabe's pullbacks of the delta functions, also known as Donsker's delta function). One interpretation of this LDP is a generalization of the LDP of Freidlin-Wentzell type for pinned diffusion measures. This LDP looks very nice. To the author's knowledge, however, no proof has been given yet.
In this talk we reformulate this LDP on the geometric rough path space by lifting these measures to the rough path sense and prove it rigorously by using quasi-sure analysis (which is a kind of potential theory in Malliavin calculus). Then, we obtain the LDP in Takanobu-Watanabe (1993) as a simple corollary of our main result. As a special case of this corollary, we also obtain the LDP for pinned diffusion measures under the strong Hörmander condition anywhere. (Even this one might be new.)

Localized upper bounds of heat kernels for diffusions via a multiple Dynkin-Hunt formula

Naotaka Kajino (Kobe University)

This talk will be devoted to presenting a recent joint work with Alexander Grigor'yan (University of Bielefeld) proving that for a general diffusion process, certain assumptions on its behavior ONLY within a FIXED open subset of the state space imply the existence and sub-Gaussian type off-diagonal upper bounds of the heat kernel on the fixed open set. The proof is mostly probabilistic and is based on a seemingly new formula, which we call a "multiple Dynkin-Hunt formula", expressing the transition function of a Hunt process in terms of that of the part process on a given open subset. This result has an application to heat kernel analysis for the Liouville Brownian motion, the canonical diffusion in a certain random geometry of the plane induced by a (massive) Gaussian free field.

Functional CLTs for non-symmetric random walks on crystal lattices

Hiroshi Kawabi (Okayama University)

In this talk, we discuss two kinds of functional CLTs for non-symmetric random walks on crystal lattices. We first show that the Brownian motion on the Euclidean space with the Albanese metric appears as the usual CLT-scaling limit. Introducing a family of random walks which interpolates between the original (non-symmetric) random walk and the symmetric one, we show that the Brownian motion with a constant drift of the asymptotic direction on the Euclidean space also appears as another kind CLT scaling limit. This is joint work with Satoshi Ishiwata (Yamagata University) and Motoko Kotani (Tohoku University).

Minimal thinness for jump processes

Panki Kim (Seoul National University)

Minimal thinness is a notion that describes the smallness of a set at a boundary point. In this talk, we discuss tests for minimal thinness at finite and infinite minimal Martin boundary points for several classes of purely discontinuous jump processes. This is a joint work with Renming Song and Zoran Vondracek.

Continuity and bounds of the density functions of the solutions to path-dependent stochastic differential equations

Seiichiro Kusuoka (Tohoku University)

We consider Markovian stochastic differential equations with low regular coefficients and the perturbation by adding a low regular path-dependent drift term. When we assume that the diffusion coefficient matrix is uniformly positive definite, then the solution to the perturbed equation is given by the Girsanov transformation of the original equation. By using the expression we obtain the Gaussian two-sided estimate and the continuity of the density function of the solution to the perturbed equation.

A dimensional Wasserstein contraction characterizing the curvature-dimension condition

Kazumasa Kuwada (Tokyo Institute of Technology)

On metric measure spaces, there are several formulations of curvature-dimension conditions, which are all equivalent to lower Ricci curvature bound and upper dimension bound on Riemannian manifolds. As one of those conditions, there is an estimate of the Wasserstein distance for heat distributions. This estimate plays a fundamental role when we extend the equivalence of such conditions on non-smooth spaces. To characterize finiteness of upper dimension bound by this condition, we had required an estimate for two heat distributions at different times.
In this talk, we introduce a new estimate of the Wasserstein distance for two heat distributions at the same time and different initial data along the approach by Bolley, Gentil and Guillin. We show that it is also equivalent to the curvature-dimension condition(s) in full generality. This is a joint work with F. Bolley, I. Gentil and A. Guillin.

Analytic characterization of gaugeability for generalized Feynman-Kac functionals and its applications

Kazuhiro Kuwae (Fukuoka University)

The talk is based on the joint work with Daehong Kim and Mila Kurniawaty. We relax the conditions for measures on the analytic characterization for gaugeability of generalized Feynman-Kac functionals. This extends the previous works on the analytic characterization with Daehong. We show that our analytic characterization of gaugeability is also equivalent to the maximum principle for our Feynman-Kac functionals provided the underlying process has strong Feller property, which extends the recent paper; The bottom of the spectrum of time-changed processes and the maximum principle of Schrödinger operators, by M. Takeda.

W-entropy formulas and rigidity theorems on Wasserstein space over Riemannian manifolds

Xiangdong Li (Chinese Academy of Science)

In this talk, we will first present the W-entropy formula for the heat equation of the Witten Laplacian on Riemannian manifolds with suitable curvature-dimension conditions or with finite dimensional Perelman's Ricci flow. Then we introduce the W-entropy and prove its monotonicity along the geodesic flow on the Wasserstein space over Riemannian manifolds. We find that these two W-entropy formulas have the same feature. This leads us to introduce a deformation of geometric flows on the Wasserstein space over Riemannian manifolds, which interpolates the geodesic flow on the Wasserstein space and the heat equation of the Witten Laplacian on the underlying manifold. We prove an entropy-energy formula along the deformation of geometric flows with a parameter c. Some rigidity theorems will be derived.

Stochastic homogenization in geometry

Xue-Mei Li (University of Warwick)

I discuss interpolation equations and their scaling limits. The equations are motivated by collapsing of Riemannian manifolds and interpolate an exponential map $\exp(Y_0)$ on a Lie group G and a hypoelliptic diffusion on its sub-group H. Assuming a reductive structure $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}$, we deduce a pair of effective stochastic processes and discuss when the effective motion on the orbits manifolds is a Markov process. The effective motion on G is classified according to the position of Y_0 in an Ad_H invariant decomposition of $\mathfrak{m} = \mathfrak{m}_0 \oplus \mathfrak{m}_1 \cdots \oplus \mathfrak{m}_r$ and can be computed from eigenvalues of the perturbation operator L_0. If $L_0 = \Delta_H$ and Y_0 in $\mathfrak{m}_l$, the effective operator on G is 'round' if the $\dim(\mathfrak{m}_l)$ is not 4 or 7. If H is a maximal torus of a semi-simple Lie group we obtain a semi-elliptic round operator. This talk is based on the following papers: arXiv:1505.06772 and arXiv:1501.04793

Stochastic Newton equation with absorbing area

Song Liang (University of Tsukuba)

We consider the motion of a particle evolving according to stochastic Newton equation with friction, with its potential given by a multiple of a compact supported function. We are interested in the limit behavior of the particle when the potential coefficient goes to infinity, in the case where the potential function gives us an absorbing force. We prove a convergence result for the distribution process of the position of the particle, with the precise expression of the limit process given.

Invariance principle for variable speed random walks on trees

Wolfgang Löhr (University of Duisburg-Essen)

The well-known path-space convergence of rescaled random walks to Brownian motion was generalised in 1963 by Stone to processes "in natural scale" on $\mathbb{R}$. These processes are characterised by a speed measure ν and include both random walks and Brownian motions with state-dependent speed. We generalise Stone's result that they depend continuously on the speed measure to the case of locally compact metric measure trees. We construct the processes via Dirichlet forms, and obtain a general continuity theorem with respect to the Gromov-Hausdorff-weak topology. This includes the convergence of random walks on Galton-Watson trees to Brownian motion on the limiting Lévy-tree, proven earlyer by Croydon with different methods. (joint work with Siva Athreya and Anita Winter)

Variational convergence on Riemannian manifolds

Jun Masamune (Tohoku University)

In this paper, we will discuss new developments in the area of variational convergence of energy functionals on Riemannian manifolds. Variational convergence such as Gamma convergence and Mosco convergence is a powerful method to derive convergences of solutions in both elliptic and parabolic equations. The theory has been developed mainly for the domains of an Euclidean space using some specific symmetric properties of the underlying space. Here, we will discuss an extension of the theory; in particular, the homogenization and singular perturbation, to Riemannian manifolds.

Spectral gap for surface diffusion

Yukio Nagahata (Niigata University)

Surface diffusion is one of the evolutional model of discrete surface. The surface is associated with two dimensional Young diagram. There is a bijection between two dimensional Young diagram and the configuration of the exclusion process. Hence we formulate this process as an exclusion process. In this talk we give a spectral gap estimate for this model.

Error analysis for approximations to one-dimensional SDEs via perturbation method

Nobuaki Naganuma (Tohoku University)

We consider one-dimensional stochastic differential equations driven by fractional Brownian motions and adopt the Euler scheme, the Milstein type scheme and the Crank-Nicholson scheme to approximate solutions to the equations. We introduce perturbation method in order to analyze errors of the schemes. By using this method, we can express the errors in terms of directional derivatives of the solutions explicitly. We obtain asymptotic error distributions of the three schemes by combining the expression of the errors and the fourth moment theorem.
This talk is based on joint work with Prof. Shigeki Aida (Tohoku University).

Phase transitions of random walk pinning model

Makoto Nakashima (University of Tsukuba)

We consider about the random walk pinning model(RWPM). This model is a random walk X on $\mathbb{Z}^d$, whose law up to time n is given as the Gibbs measure with weight $e^{\beta L_n(X,Y)}$ conditioned on the trajectory of Y, where Y is an independent random walk on $\mathbb{Z}^d$ and $L_n(X,Y)$ is the collision local time of X and Y up to time n. It is known that there exist two types of definition of phase transitions, described in terms of the partition functions and in terms of the free energies. We will talk about the coincidence of these critical points.

A regularity theory for elliptic systems with random coefficients

Stefan Neukamm (Technical University of Dresden)

We study the regularity of elliptic systems with stationary and ergodic, random coefficients and consider the associated quantitative stochastic homogenization problem. Following the philosophy of Avellaneda and Lin, developed in the context of deterministic, periodic homogenization, we lift the regularity theory of the homogenized limit to the heterogeneous situation and obtain $C^{1,\alpha}$ estimates on large scales, where large is quantified in terms of a minimal radius that measures the (sublinear) growth of the generalized corrector. Based on a quantification of ergodicity (which allows for environments with arbitrarily slow-decaying correlations), we derive stretched exponential moment bounds for the corrector and obtain various quantitative homogenization results, e.g. on the spatial growth of the corrector and the error of the two-scale expansion. The talk is based on a joint work with Antoine Gloria and Felix Otto.

Modified Arratia flow and Wasserstein diffusion

Max von Renesse (University of Leipzig)

We introduce a modified Arratia flow of sticky Brownian motions with masses under an additional preservation of total diffusivity of the full system. The induced measure valued flow is a weak solution to the SPDE of the Wasserstein diffusion but with a modified drift. Moreover, its large deviations in the short time asymptotics are given by the Wassserstein distance.

Infinite dimensional continuity and Fokker-Planck-Kolmogorov equations

Michael Röckner (University of Bielefeld)

We present a new uniqueness result for solutions to Fokker-Planck-Kolmogorov (FPK) equations for probability measures on infinite-dimensional spaces. We consider infinite-dimensional drifts that admit certain finite dimensional approximations. In contrast to most of the previous work on FPK-equations in infinite dimensions, we include cases with non-constant coefficients in the second order part and also include degenerate cases where these can even be zero, i.e. we prove uniqueness of solutions to continuity equations. Also new existence results are proved. Applications to proving well-posedness of Fokker-Planck-Kolmogorov equations associated with SPDEs and of continuity equations associated with PDE are discussed. This is joint work with Vladimir Bogachev, Giuseppe Da Prato and Stanislav Shaposhnikov

Long-time behavior of Lévy-type processes: transience, recurrence and ergodicity

Nikola Sandrić (Technical University of Dresden and University of Zagreb)

In this talk, we discuss transience, recurrence and ergodicity of Feller processes associated with pseudo-differential operators —the so-called Lévy-type processes. First, we present Chung-Fuchs type conditions for the transience and recurrence in terms of the symbol of the corresponding pseudo-differential operator. Next, by using these conditions, we discuss the transience and recurrence with respect to the dimension of the state space and Pruitt indices and the transience and recurrence of Feller-Dynkin diffusions and stable-like processes. In the one and two-dimensional cases, we discuss perturbations of Feller processes which do not affect their transience and recurrence properties, and we present sufficient conditions for their transience and recurrence in terms of the corresponding Lévy measure. At the end, we discuss strong, polynomial and exponential ergodicity of Feller processes.

The logarithmic Sobolev inequality and the convergence of a semigroup in the Zygmund space

Ichiro Shigekawa (Kyoto University)

It is well-known that the logarithmic Sobolev inequality implies the exponential convergence of the entropy. In this talk, we connect this fact to the convergence in the Zygmund space $L \log L$. In fact, we can show the exponential convergence of a semigroup in the Zygmund space under the assumption of the logarithmic Sobolev inequality.
We also give an example of the spectrum of the generator in the Zygmund space.

Escape rate of symmetric Markov processes

Yuichi Shiozawa (Okayama University)

We are concerned with the global path properties of symmetric Markov processes generated by regular Dirichlet forms. In particular, we characterize these properties in terms of the upper and lower rate functions. The upper rate function, which is a quantitative version of conservativeness, expresses forefront of particles for all sufficiently large time. On the other hand, the lower rate function, which is a quantitative version of transience, expresses the speed of particles escaping to infinity. We discuss about the characterizations of these functions in terms of the volume and coefficient growth rates.

Optimal transport, Brownian motion, and super-Ricci flow for metric measure spaces

Karl-Theodor Sturm (University of Bonn)

We study heat equation and Brownian motion on time-dependent metric measure spaces with particular emphasis on mm-spaces which evolve as super-Ricci flows. A time-dependent family of Riemannian manifolds is a super-Ricci flow if $2\mathrm{Ric} + \partial_t g \geq 0$. This includes all static manifolds of nonnegative Ricci curvature as well as all solutions to the Ricci flow equation.
We extend this concept of super-Ricci flows to time-dependent metric measure spaces. In particular, we present characterizations in terms of dynamical convexity of the Boltzmann entropy on the Wasserstein space as well in terms of Wasserstein contraction bounds and gradient estimates for the heat equation. And we prove stability and compactness of super-Ricci flows under mGH-limits.

Random walks on hyperbolic groups: entropy and speed

Ryokichi Tanaka (Tohoku University)

Associated with random walks on groups, we consider three fundamental quantities: entropy, speed and volume growth (exponential growth rate of the group). The fundamental inequality due to Guivarc'h tells that the entropy does not exceed the speed times the volume growth. Vershik (2000) asked about the genuine equality case. We focus on hyperbolic groups, and characterise the equality case; namely, the equality holds if and only if the harmonic measure and a natural measure–the Hausdorff measure–on the boundary are equivalent. We also present some open problems related to this question. We start with a history of this problem and mention about recent progresses as well. All the notion will be explained during the talk.

Systems of infinitely many Brownian motions with long ranged interaction

Hideki Tanemura (Chiba University)

We study infinite dimensional stochastic differential equations (ISDEs) describing the dynamics of infinitely many Brownian motions moving in $\mathbb{R}^d$ with free potential Φ and mutual interaction Ψ. We show existence and uniqueness of solution of the ISDES for essentially all interaction potential of Ruelle's class and logarithmic potentials.

Recurrence criteria for diffusion processes generated by divergence free perturbations of non-symmetric energy forms

Gerald Trutnau (Seoul National University)

On a metric space E, we consider a generalized Dirichlet form

$$\mathcal{E}(f,g) = \mathcal{E}^0(f,g) + \int_E fNg\, d\mu,$$

where $(\mathcal{E}^0, D(\mathcal{E}^0))$ is a sectorial Dirichlet form on $L^2(E,d\mu)$, $(N, D(N))$ is a linear operator on $L^2(E,d\mu)$ and $\mathcal{E}^0(f,f) \leq \mathcal{E}(f,f)$. We find a criterion for recurrence of $\mathcal{E}$. Namely, if the generalized Dirichlet form $\mathcal{E}$ is strictly irreducible and if there exists a sequence of functions $(\chi_n)_{n\geq 1}$ with $0 \leq \chi_n \leq 1$, $\lim_{n\to\infty} \chi_n = 1$ μ-a.e. satisfying

$$\lim_{n\to\infty} \left(\mathcal{E}^0(g, \chi_n) + \int_E gN\chi_n\, d\mu \right) = 0 \tag{1}$$

for any non-negative bounded g in the extended Dirichlet space of $D(\mathcal{E}^0)$, then $\mathcal{E}$ is recurrent. As application, we consider $E \subset \mathbb{R}^d$, E open or closed and a strictly irreducible generalized Dirichlet form

$$\mathcal{E}(f,g) = \frac{1}{2}\int_E A(\nabla f)\cdot \nabla g\, d\mu - \int_E (B\cdot \nabla f) g\, d\mu,$$

where the diffusion matrix $A = (a_{ij})$ is not necessarily symmetric but its antisymmetric part consists of bounded functions and B is a locally μ-square integrable μ-divergence free vector field. Then using volume growth conditions of B and of the sectorial part of $\mathcal{E}$ on Euclidean balls w.r.t. μ, we construct explicitly $(\chi_n)_{n\geq 1}$ satisfying (1). One astonishing observation is that there may exist a sequence of functions $(\chi_n)_{n\geq 1}$ in the Dirichlet space with $0 \leq \chi_n \leq 1$, $\lim_{n\to\infty} \chi_n = 1$ μ-a.e. and such that $\lim_{n\to\infty} \mathcal{E}(\chi_n, \chi_n) = 0$, but recurrence does not hold for $\mathcal{E}$ in contrast to the symmetric case. We consider several examples and counterexamples when μ is equivalent to Lebesgue measure with concrete density $\mu = \rho\, dx$. This is joint work with Minjung Gim (Seoul National University).

Large time asymptotics of Feynman-Kac functionals for symmetric α-stable processes

Masaki Wada (Tohoku University)

Let $\{X_t\}_{t\geq 0}$ be a transient Hunt process on $\mathbb{R}^d$ with generator $(-\Delta)^{\frac{\alpha}{2}}$ $(0 < \alpha < 2)$. Suppose μ is a positive Radon smooth measure in a certain class and denote by A_t^μ the positive continuous additive functional in the Revuz correspondence with μ. In this talk, we give the asymptotic behavior of $\mathbb{E}_x[\exp(A_t^\mu)]$ as $t \to \infty$ for critical μ. We compare with the preceding result by Cranston et al where the same problem is studied for the Brownian motion, i.e. the stable process with $\alpha = 2$. Similarly to the Brownian motion, the growth order of $\mathbb{E}_x[\exp(A_t^\mu)]$ depends on d/α. However, our method of the proof is different from theirs in that we mainly use the Dirichlet form theory for time-changed processes by μ. This is a jointly work with Professor Masayoshi Takeda.

Abstracts of the Poster session

Maximum and minimum of local times for two-dimensional random walk

Yoshihiro Abe (Kyoto Univ.)

I will describe the leading orders of the maximum and the minimum of local times for the simple random walk on the two-dimensional discrete torus at time comparable to the cover time. I also discuss the size of level sets close to the maximum and the minimum of local times. These are closely related to estimates on the two-dimensional Gaussian free fields by Bolthausen-Deuschel-Giacomin (2001) and Daviaud (2006), and I have different exponents from the case of the Gaussian free field.

Convergence implications via dual flow method

Takafumi Amaba (Ritsumeikan Univ.)

Given a sequence of discrete time stochastic flow $\{X^n_{k,l}\}_{k\leq l}$, $n \in \mathbb{N}$ as an approximation of a stochastic flow $\{X_{s,t}\}_{s\leq t}$ on $[0,+\infty)$, we will see how the rate of convergence for their dual flows is implied from the rate of convergence of $\{X^n_{k,l}\}_{k\leq l} \to \{X_{s,t}\}_{s\leq t}$.

Infinite particle systems of long range jumps with long range interactions

Syota Esaki (Tokyo Inst. Tech.)

We present general theorem of constructing infinite particle systems of jump type with long range interaction by the Dirichlet form technique. Our theorem includes the case that each particles undergoes α-stable processes and interaction between particles given by the logarithmic potential appearing random matrix theory and potentials of Ruelle's class with polynomial decay. In the assumption a jump rate is restricted by the 1-correlation function of the equilibrium measure. This is necessary for infinitely particles not to concentrate on any compact set. In particular this assumption is satisfied for any $\alpha \in (0,2)$ in case a equilibrium measure is translation invariant.

Disease spreading models within the framework of two-component configuration spaces in continuum

Torben Fattler (TU Kaiserslautern)

We present the mathematical modeling of the spread of an infectious disease within the framework of interacting particle systems in continuum. Starting with the Markov generator of an underlying stochastic process in a two-component configuration space, it is natural to study the time evolution of states. I.e., we analyze the time evolution of probability measures on a two-component configuration space in continuum. Technically, we consider the corresponding chain of hierarchical equations for correlation functions. Our goal is to obtain macroscopic Vlasov-type equations from the microscopic infinite particle system, describing the evolution of the density of the evolving initial state of the system modeling the spread of an infectious disease. The approach is based on the fundamental idea that dynamical processes in many-body systems are often approximately described by kinetic equations.

Unbounded viscosity solutions of non-dominated HJB-equations

Julian Hollender (TU Dresden)

Hamilton-Jacobi-Bellman (HJB) equations of the form

$$\partial_t u(t,x) + \sup_{\alpha\in\mathcal{A}} G_\alpha u(t,x) = 0$$

for a family of linear 2nd order integro-differential operators $(G_\alpha)_{\alpha\in\mathcal{A}}$ play a central role in stochastic optimal control theory with respect to jump-diffusions. The closely related field of G-Lévy processes (as introduced by Mingshang Hu and Shige Peng in 2009) naturally leads to HJB-equations, where the related family of jump measures cannot be dominated by a single jump measure. Consequently, the corresponding operators are (in general) not continuous with respect to pointwise but only local uniform convergence. We will establish a maximum and comparison principle (and therefore the uniqueness for the associated initial value problem) for (viscosity) solutions with arbitrary polynomial growth for such equations.

KPZ equation with fractional derivatives of white noise

Masato Hoshino (Univ. of Tokyo)

We discuss the stochastic partial differential equation

$$\partial_t h(t,x) = \partial_x^2 h(t,x) + (\partial_x h(t,x))^2 + \partial_x^\gamma \xi(t,x)$$

for $(t,x) \in [0,\infty) \times \mathbb{T}$ with $\gamma \geq 0$.

Theorem 1 *Let $\rho \in C_0^\infty(\mathbb{R}^2)$ be a mollifier with nice properties. Set $\rho_\epsilon(t,x) = \epsilon^{-3}\rho(\epsilon^{-2}t, \epsilon^{-1}x)$ and $\xi_\epsilon = \xi * \rho_\epsilon$. If $0 \leq \gamma < \frac{1}{4}$, then there exists a sequence of constants $C_\epsilon = \mathcal{O}(\epsilon^{-1-2\gamma})$, and the sequence of solutions h_ϵ to the equation*

$$\partial_t h_\epsilon(t,x) = \partial_x^2 h_\epsilon(t,x) + (\partial_x h_\epsilon(t,x))^2 - C_\epsilon + \partial_x^\gamma \xi_\epsilon(t,x)$$

for $(t,x) \in [0,\infty) \times \mathbb{T}$ converges to a unique stochastic process h, which is independent of the choice of ρ.

Integration of controlled rough paths via fractional calculus

Yu Ito (Osaka Univ.)

This study is an alternative approach to the fundamental theory of rough paths on the basis of fractional calculus. In this talk, using fractional derivatives, we will introduce integration of controlled rough paths. This can be regarded as an extension of the integration introduced by Y. Hu and D. Nualart in 2009. This can also be regarded as an extension of the integration introduced in the speaker's previous works. As an application, we will provide an explicit expression of the second level path of the rough integrals against geometric β-Hölder rough paths with $\beta \in (1/3, 1/2]$.

Finite particle approximation of interacting Brownian motion

Yosuke Kawamoto (Kyushu Univ.)

We consider infinite-dimensional SDE (ISDE) describing interacting Brownian motion with infinitely many particles. We prove a general theory of finite particle approximation. This theory gives a convergence from a solution of finite-dimensional SDE to that of ISDE. Furthermore, our results can be applied to many models, for example, related to random matrices, interacted Lennard-Jones potential, and so on.

Heat kernel estimates for symmetric Markov processes in $C^{1,\rho}$ open sets and its application

Kyung-youn Kim (Seoul Nat. Univ.)

We consider a large class of symmetric Markov processes dominated by isotropic unimodal Lévy processes with weak scaling conditions. We first establish sharp two-sided heat kernel estimates for this processes in $C^{1,\rho}$ open sets, $\rho \in (\overline{\alpha}/2, 1]$. As a corollary of our main result, we obtain a sharp two-sided Green function and a global boundary Harnack inequality with explicit decay rates in $C^{1,\rho}$ open sets.

Minimizing movement scheme for time dependent metrics on probability space

Eva Kopfer (Univ. of Bonn)

Let $(g_t)_{t\in[0,T]}$ be a family of Riemannian metrics on a smooth connected manifold M such that (M, g_t) is complete for every t. We study the time dependent notion of the gradient flow for the relative entropy on probability measures over M via the minimizing movement scheme. This gradient flow will coincide with the conjugate heat flow.

Existence and estimates of moments for Lévy-type processes

Franziska Kühn (TU Dresden)

We present recent results on the existence of moments and moment estimates for Lévy-type processes. The following questions are discussed: Is the existence of moments a time dependent distributional property? What are sufficient conditions for the existence of moments? How to estimate fractional moments? For the class of Lévy processes the answers are well-known, but up to the present there are only few general results for Lévy-type processes. Our results apply in particular to SDEs and stable-like processes (stable processes with varying index of stability) and can be used to prove integrated heat kernel estimates for Lévy-type processes.

Pathwise uniqueness for SDEs with non-regular drift and non-constant diffusion

Katharina von der Lühe (Univ. of Bielefeld)

A new approach to prove pathwise uniqueness for SDEs of the form

$$dX_t = b(t, X_t)dt + dW_t$$

was introduced by E. Fedrizzi and F. Flandoli in [1]. We generalize this method to SDEs with time and space dependent diffusion

$$dX_t = b(t, X_t)dt + \sigma(t, X_t)dW_t,$$

where the matrix-valued function σ is non-degenerated, bounded, continuous and its weak derivative $\partial_x \sigma$ as well as σ is in L_p^q. The proof is based on a transformation via solutions to PDEs of the form

$$\partial_t u + \frac{1}{2} \sum_{i,j} (\sigma\sigma^*)_{ij} \partial_{x_i} \partial_{x_j} u = -b.$$

References

[1] Fedrizzi, E. and Flandoli, F., *Pathwise uniqueness and continuous dependence of SDEs with non-regular drift*, Stochastics. **83** (2011), 241–257.

The conservativeness of Girsanov transformed symmetric Markov processes

Yusuke Miura (Tohoku Univ.)

We consider Girsanov transformations of symmetric Markov processes which preserve the symmetry. Employing a criterion for uniform integrability of exponential martingales due to Chen, we identify the class of transformations which transform the original process into a conservative one, even if the original one is explosive. We also show the recurrence property of transformed processes. There exists previous work in which the same problems are considered for symmetric diffusion processes. Our result is an extension to symmetric Markov processes with jumps.

Lamplighter random walks on fractals

Chikara Nakamura (Kyoto Univ.)

Lamplighter graph is a graph which has lamps on each vertex of the graph, and we take into account which lamps are on/off. For the lamplighter graphs, number of vertecies which belong to balls grows exponentially fast with respect to the radius of the ball, and there are no general methods to analyze random walk on such graphs. So we investigated random walks on the lamplighter graphs.
In this poster presentation, I will present some works on the lamplighter graphs. We considered fractal lamplighter graphs and obtained the sharp on-diagonal heat kernel estimates and the law of the iterated logarithms for the random walks on these graphs. This is a joint work with Prof. T. Kumagai.

Geometry structures of favorite sites of random walk range

Izumi Okada (Tokyo Inst. Tech.)

We consider the question: how many times does a simple random walk revisit the most frequently visited site among the inner boundary points? It is known that the number of visits to a most frequently visited site among all of the points has the phase transition between two dimension and higher dimensions. On the other hand, we prove that the corresponding number among the inner boundary does not has it. In addition, we can state that the most frequently visited site among all of the points does not appear in the inner boundary from some time on with probability one.

Convergence of Brownian motions on $\mathrm{RCD}^*(K,N)$ spaces

Kohei Suzuki (Kyoto Univ.)

Suppose that a sequence of metric measure spaces (X_n, d_n, m_n) satisfies $\mathrm{RCD}^*(K,N)$ with Diam $\leq D$ and $m_n(X_n) = 1$. Then we show that the measured Gromov–Hausdorff convergence of (X_n, d_n, m_n) is equivalent to the weak convergence of the laws of Brownian motions on (X_n, d_n, m_n).

Interacting particle systems with sticky boundary

Robert Voßhall (TU Kaiserslautern)

We present the construction of a system of interacting particles in a bounded domain with sticky boundary, i.e., the reflection of the particles at the boundary is delayed. Additionally, an optional diffusion along the boundary is provided. Moreover, we analyze the constructed process and identify it as solution to the associated SDE. Finally, we discuss applications with singular interactions. Based on a joint work with Martin Grothaus.

Diffusion determines the recurrent graph

Melchior Wirth (Friedrich-Schiller-Universität Jena)

We discuss properties of weighted graphs that are encoded in the graph Laplacian. Analogous to the work of Wolfgang Arendt for Laplacians on domains in Euclidean space, we study graphs for which the semigroups of the graph Laplacians are equivalent up to an order isomorphism. We show that the order isomorphism is automatically unitary (up to a constant) in this situation and present some geometric quantities that are preserved. For important classes of graphs, all defining properties can be recovered (joint work with Matthias Keller, Daniel Lenz, and Marcel Schmidt).

Central limit theorem for stochastic heat equations in random environments

Lu Xu (Univ. of Tokyo)

In this poster, we proved the central limit theorem (CLT) for stochastic heat equations with random coefficients, as an infinite-dimensional extension of the same study on diffusion processes in random environments. We considered a 1-dimensional stochastic heat equation driven by a space-time white noise, and adopted the nonlinear term as a stationary and ergodic function-valued random field. We proved the CLT for its mild solution, with the limit distribution degenerating to a 1-dimensional Gaussian law. Our study involves the construction of the environmental processes in infinite-dimensional settings, as well as an careful check on conditions necessary for applying the functional CLT for general Markov processes and Itô formula.