Marginal density expansions for diffusions and stochastic volatility
Jean-Dominique Deuschel
TU Berlin

Abstract: Density expansions for hypoelliptic diffusions \((X^1, \ldots, X^d)\) are revisited. In particular, we are interested in density expansions of the projection \((X^{1,T}, \ldots, X^{l,T})\), at time \(T > 0\), with \(l \leq d\). Global conditions are found which replace the well-known "not-in-cutlocus" condition known from heat-kernel asymptotics; cf. G. Ben Arous (1988). Our small noise expansion allows for a "second order" exponential factor. Applications include tail and implied volatility asymptotics in some correlated stochastic volatility models; in particular, we solve a problem left open by A. Gulisashvili and E.M. Stein (2009).

This is a joint work with P. Friz, A. Jacquier and S. Violante.
Tunneling for spatially cut-off $P(\phi)_2$-Hamiltonians
Shigeki Aida
Tohoku University

Abstract: We will talk about asymptotic behaviors of low-lying spectrum of $P(\phi)_2$-Hamiltonian $-L + V_\lambda$ in the semi-classical limit $\lambda \to \infty$. The corresponding classical equation of the $P(\phi)_2$-field is a nonlinear Klein-Gordon equation which is an infinite dimensional Newton’s equation. We determine the semi-classical limit of the lowest eigenvalue of the spatially cut-off $P(\phi)_2$-Hamiltonian in terms of the Hessian of the potential function of the Klein-Gordon equation. Moreover, we show that the gap of the lowest two eigenvalues goes to 0 exponentially fast in the semi-classical limit when the potential function is double well type. In fact, we prove that the exponential decay rate is greater than or equal to the Agmon distance between two zero points of the symmetric double well potential function. The Agmon distance is a Riemannian distance on the Sobolev space $H^{1/2}(\mathbb{R})$ defined by a Riemannian metric which is formally conformal to $L^2$-metric. Also we will talk about basic properties of the Agmon distance and instanton.

References

Abstract: Random fields of gradients are a class of model systems arising in the studies of random interfaces, random geometry, field theory, and elasticity theory. These random objects pose challenging problems for probabilists as even an a priori distribution involves strong correlations. Gradient fields are likely to be an universal class of models combining probability, analysis and physics in the study of critical phenomena. They emerge in the following three areas, effective models for random interfaces, Gaussian Free Fields (scaling limits), and mathematical models for the Cauchy-Born rule of materials, i.e., a microscopic approach to nonlinear elasticity. The latter class of models requires that interaction energies are non-convex functions of the gradients. Open problems include unicity of Gibbs measures and strict convexity of the free energy. We present in the talk a first breakthrough for the free energy at low temperatures using Gaussian measures and rigorous renormalization group techniques. In addition we show that the correlation functions in that regime have Gaussian decay properties despite a non-convex interaction potential. The key ingredient is a finite range decomposition for parameter dependent families of Gaussian measures.
Hydrodynamic limit for the Ginzburg-Landau $\nabla \phi$ interface model with both a conservation law and the Dirichlet boundary condition

Takao Nishikawa
Nihon University

Abstract: We discuss the hydrodynamic scaling limit for the interface model with a conservation law, that is, the stochastic dynamics of the interface preserving its volume. For the dynamics on the torus, that is, the dynamics under the periodic boundary condition, the nonlinear fourth-order partial differential equation is derived as the macroscopic equation in [N. 2002]. The aim of this talk is to discuss the behavior of the interface motion with both the conservation law and the Dirichlet boundary condition, and to derive the macroscopic equation. We also discuss the relationship with the Wulff shape derived by [Deuschel-Giacommin-Ioffe, 2001].
Exponential convergence of Markovian semigroups and their spectra on $L^p$-spaces
Seiichiro Kusuoka
Kyoto University

Abstract: A Markovian semigroup on $L^2$-space with suitable conditions can be regarded as a Markovian semigroup on $L^p$-spaces for $p \in [1, \infty)$. When we additionally assume ergodicity of the Markovian semigroup, the rate of convergence can be considered. However, generally the rate of convergence is closely depends on the norm of the spaces. The purpose of this paper is to investigate the relation between the rates with respect to $L^p$-spaces with different $p$, to obtain some sufficient condition for the rates to be independent of $p$, and to give an example that the rates depend on $p$. We also consider spectra of Markovian semigroups with respect to $L^p$-spaces, because the rate of convergence is closely related to the spectra.
Abstract: We study the freely infinitely divisible distributions that appear as the laws of free subordinators. This is the free analog of classically infinitely divisible distributions supported on $[0, \infty)$, called the free regular infinitely divisible measures.

We prove that the class of free regular measures is closed under the free multiplicative convolution, $t$th boolean power for $0 \leq t \leq 1$, $t$th free multiplicative power for $t \geq 1$ and weak convergence.

In addition, we show that a symmetric distribution is freely infinitely divisible if and only if its square can be represented as the free multiplicative convolution of a free Poisson and a free regular measure.

This gives two new explicit examples of distributions which are infinitely divisible with respect to both classical and free convolutions: $\chi^2$ and $F(1,1)$. Another consequence is that the free commutator operation preserves free infinite divisibility. This is joint work with T.Hasebe(Kyoto,Japan) and O. Arizmendi(Saalandes,Germany).
Regularity theory for parabolic nonlocal operators
Moritz Kassmann
Bielefeld University

Abstract: Using the framework of Dirichlet forms we study weak solutions to parabolic nonlocal problems. We prove Hölder-regularity and a Harnack-type inequality for weak solutions to parabolic equations with integro-differential operators of fractional order. Assumptions on the kernel are discussed in detail and several examples are provided. One feature of our result is robustness with respect to the order of differentiability of the generator.

This work is joint with M. Felsinger.
Essential spectra and volume growth of regular Dirichlet forms
Matthias Keller
University of Jena

Abstract: In Riemannian geometry the exponential volume growth yields an upper bound for the bottom of the essential spectrum. In contrast, there are graphs of polynomial growth that already have positive bottom of the essential spectrum, if the volume is measured with respect to the natural graph distance. This disparity can be resolved in the common framework of regular Dirichlet forms. There, one has the concept of intrinsic metrics for strongly local forms which was recently extended systematically to all regular Dirichlet forms. If the volume is measured with respect to such an intrinsic metric, then the classical result can be achieved and improved in great generality. Moreover, for graphs one can relate the natural graph distance to a special intrinsic metric. In this way one finds that the threshold for positive bottom of the essential spectrum lies at cubic growth with respect to the natural graph distance. (This is joint work with Sebastian Haeseler and Radosław Wojciechowski.)
Infinitely dimensional stochastic equations related to Airy random point fields
Hirofumi Osada
Kyushu University

Abstract:
Stochastic completeness of jump processes and random walks
Xueping Huang
Bielefeld University

Abstract: A jump process on some metric measure space is called stochastically complete (or conservative) if the process does not die out in finite time. It is interesting to relate this long time behavior of the process to the large scale geometry of the underlying space. In this talk, we report on some recent progress in this direction. A particular case, the continuous time random walks on weighted graphs, will be discussed in details.
Perturbation of Dirichlet forms and stability of fundamental solutions

Masaki Wada
Tohoku University

Abstract: Let \((E, \mathcal{F})\) be a symmetric regular Dirichlet form as follows:

\[
E(u, v) = \int_{\mathbb{R}^d \times \mathbb{R}^d} (u(y) - u(x))(v(y) - v(x))J(x, y)dxdy
\]

\[
\frac{\kappa_1}{|x-y|^d\phi(|x-y|)} \leq J(x, y) \leq \frac{\kappa_2}{|x-y|^d\phi(|x-y|)} \quad (0 < \kappa_1 \leq \kappa_2)
\]

\[
\phi(r) = r^\alpha \exp(m(r-1) \lor 0) \quad (0 < \alpha < 2, m \geq 0)
\]

It is proved that the associated fundamental solution \(p(t, x, y)\) admits the two-sided estimates by Chen, Kumagai et al. Let \(\mu\) be a positive Radon smooth measure in Kato class and consider the perturbed form \(E(\mu)(u, u) = E(u, u) - \int_{\mathbb{R}^d} u(x)d\mu\). Denote the fundamental solution associated with \(E(\mu)\) by \(p^\mu(t, x, y)\). We call it the stability of fundamental solution that \(p^\mu(t, x, y)\) has the same two-sided estimates as \(p(t, x, y)\) up to positive constants. In this talk, we establish the necessary and sufficient conditions for the stability.
Free boundary problems, viscosity solutions and applications in finance
Harald Oberhauser
TU Berlin

Abstract: Motivated by recent progress concerning the model impact for the pricing of exotic derivatives we revisit the classic Skorohod embedding problem for continuous Markov processes. Certain constructions of solutions to the embedding problem can in fact be naturally linked with free boundary problems (especially, parabolic obstacle problems involving the generator of the Markov process used for the embedding). We propose an approach with viscosity theory which allows for efficient and intuitive proofs; further this opens the door to stable numerical schemes.
Brownian motion with darning applied to KL and BF equations for planar slit domains
Masatoshi Fukushima
Osaka University

Abstract: A Brownian motion with darning (BMD in abbreviation) is a diffusion process on a planar region with many holes that behaves like a reflecting Brownian motion but by regarding each hole as a one-point set. We apply BMD to the study of a family of conformal mappings from a standard slit domain with growing Jordan arcs being removed onto standard slit domains. The family satisfies an equation first derived by Y. Komatu in 1950 and then by R.O. Bauer and R.M. Friedrich in 2008, extending the well known Loewner differential equation for simply connected domains. However the Komatu-Loewner differential equation has been established only in the sense of the left derivative.

It can be shown that the kernel appearing in the KL equation is just a complex Poisson kernel of BMD and the stated conformal maps are expressed probabilistically in terms of BMD. Combining these properties with a PDE method of variations of the classical Green function, we show that the KL equation is a genuine differential equation. We further derive a differential equation for the induced motion of the slits first observed by Bauer-Friedrich and thereby recover the original family of conformal maps from a given motion on the boundary. It serves as a base toward the study of SLE for multiply connected domains.

This is a joint work with Zhen-Qing Chen and Steffen Rohde.
Integrals along rough paths via fractional calculus

Yu Ito
Kyoto University

Abstract: In this talk, we will provide the definition of integrals along $\beta$-Hölder rough paths for $\beta \in (0,1]$ based on the fractional calculus. This integral is consistent with the integrals in the context of rough path analysis and is a generalization of the preceding study of Hu and Nualart, who considered the case of $\beta \in (1/3,1/2)$. Under some conditions on the integrand, we will show that this integral is a continuous functional with respect to the Hölder topology.
Annealed Brownian motion in a heavy tailed Poissonian potentials
Ryoki Fukushima
Kyoto University

Abstract: Consider the $d$-dimensional Brownian motion in a random potential defined by attaching a non-negative and polynomially decaying potential around Poisson points. We introduce a repulsive interaction between the Brownian path and the Poisson points by weighting the measure by the Feynman-Kac functional. Under the (annealed) weighted measure, it is shown that the Brownian motion tends to localize around the origin and the properly scaled process converges in law to a Ornstein-Uhlenbeck process.
Wasserstein contractions associated with the curvature-dimension condition

Kazumasa Kuwada
Ochanomizu University

Abstract: We obtain a new characterization of complete Riemannian manifolds with lower Ricci curvature bound and upper dimension bound in terms of the Wasserstein distance between heat distributions. It is formulated as a space-time Lipschitz estimate of the Wasserstein distance between two heat distributions with different initial data at different times. It extends a part of result by von Renesse and Sturm which studies the same problem without upper dimension bound. The proof is based on the study of the relation with a gradient estimate of heat semigroups recently introduced by F.-Y. Wang. In addition, we can obtain a sharper estimate by using a coupling method.
Global properties of Dirichlet forms in terms of Green’s formula
Marcel Schmidt
University of Jena

Abstract:
Lévy measure density corresponding to inverse local time

Tomoko Takemura

Nara Women’s University

Abstract: We are concerned with Lévy measure density corresponding to the inverse local time at the regular end point for harmonic transform of a one dimensional diffusion process. We show that the Lévy measure density is represented as a Laplace transform of the spectral measure corresponding to an original diffusion process, where the absorbing boundary condition is posed at the end point if it is regular.
Stochastic variational inequalities and applications to the total variation flow perturbed by linear multiplicative noise

Michael Röckner
Bielefeld University

Abstract: In this work, we introduce a new method to prove the existence and uniqueness of a variational solution to the stochastic nonlinear diffusion equation

$$dX(t) = \text{div} \left[ \frac{\nabla X(t)}{|\nabla X(t)|} \right] dt + X(t)dW(t) \text{ in } (0, \infty) \times \mathcal{O},$$

where $\mathcal{O}$ is a bounded and open domain in $\mathbb{R}^N$, $N \geq 1$, and $W(t)$ is a Wiener process of the form $W(t) = \sum_{k=1}^{\infty} \mu_k \epsilon_k \beta_k(t)$, $\epsilon_k \in C^2(\bar{\mathcal{O}}) \cap H^1_0(\mathcal{O})$, and $\beta_k$, $k \in \mathbb{N}$ are independent Brownian motions. This is a stochastic diffusion equation with a highly singular diffusivity term and one main result established here is that, for all initial conditions in $L^2(\mathcal{O})$, it is well posed in a class of continuous solutions to the corresponding stochastic variational inequality. Thus one obtains a stochastic version of the (minimal) total variation flow. The new approach developed here also allows to prove the finite time extinction of solutions in dimensions $1 \leq N \leq 3$, which is another main result of this work.

This is a joint work with Viorel Barbu (Octav Mayer Institute of Mathematics (Romanian Academy))

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On spectral bounds for symmetric Markov chains with coarse Ricci curvatures
Kazuhiro Kuwae
Kumamoto University

Abstract: I will talk about an upper estimate of non-linear (including linear) spectral radius for (non-linear) transition operator over $L^p$-maps in terms of symmetric Markov chains on a Polish space with positive $n$-step coarse Ricci curvature lower bound. The target space is a 2-uniformly convex space having some geometric conditions including the case of CAT(0)-spaces. As consequences, strong $L^p$-Liouville property for harmonic maps, global Poincaré inequality, that is, spectral gaps for energy functional over $L^2$-maps (or functions) in terms of such Markov chains, and spectral bounds of $L^2$-generator of Markov chains are presented. This is a joint work with E. Kokubo, who was my student in master course last year.

References

Geodesic distances and intrinsic distances on some fractal sets
Masanori Hino
Kyoto University

Abstract: The off-diagonal Gaussian asymptotics of the heat kernel density associated with local Dirichlet form is often described by using the intrinsic distances (or Carnot–Caratheodory distances). When the underlying space has a Riemannian structure, the geodesic distance is defined as well, which coincides with the intrinsic distance in good situations. In this talk, I will consider the case when the underlying space has a fractal structure and provide some partial results on the relations between these distances.
Well-Posedness of SPDE with Local Monotonicity and Generalized Coercivity Conditions
Wei Liu
Bielefeld University

Abstract: In this talk we will present some recent extensions on the classical result of Krylov and Rozovskii for the well-posedness of SPDE. More precisely, by replacing the standard monotonicity and coercivity conditions with some local monotonicity and generalized coercivity/Lyapunov conditions we obtain the local/global existence and uniqueness of strong solutions for a large class of SPDEs. The main results are applied to e.g. stochastic (tamed) Navier-Stokes equations, stochastic surface growth model, stochastic power law fluids and stochastic (generalized) curve shortening flow.

This talk is based on some joint works with Michael Röckner.
Super-Brownian motion in random environment and heat equation with noise.

Makoto Nakashima
University of Tsukuba

Abstract: Konno and Shiga proved that one dimensional super-Brownian motion is absolutely continuous with respect to Lebesgue measure almost surely under some initial condition and it satisfies that the heat equation with noise,

\[ u_t = \frac{1}{2} u_{xx} + \sqrt{u} \dot{W}, \]

where \( \dot{W} \) is time-space white noise. In this talk, we construct some super-Brownian as a limit point of branching random walks in random environment which is absolutely continuous with respect to Lebesgue measure almost surely and it is a solution of the heat equation,

\[ u_t = \frac{1}{2} u_{xx} + \sqrt{u + u^2} \dot{W}. \]
Quenched invariance principle for random walks and random divergence forms in random media on cones

Takashi Kumagai
Kyoto University

**Abstract:** We will consider the following two models and establish quenched invariance principles;

1. Simple random walks on the infinite clusters for super-critical percolations on half and quarter planes in d-dimensional Euclidean spaces.
2. Uniform elliptic divergence forms with random stationary coefficients on cones in Euclidean spaces.

Note that because of the lack of translation invariance, we cannot apply the method of the ‘corrector’. Instead, we make full use of the heat kernel estimates and Dirichlet form techniques to resolve the problem. This is a joint work with Z.Q. Chen (Seattle) and D.A. Croydon (Warwick).
Some path properties of Lévy-type processes
René L. Schilling
TU Dresden

Abstract: We show some path properties for Lévy-type processes using the symbol of the infinitesimal generator.