Modeling and Robust Attitude Control of Stationary Self-sustaining Two-wheeled Vehicle

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Abstract: Stability of Two-wheeled vehicles depends on their running speed. The running vehicle at high speed is stable but the vehicle in a state of stillness is unstable. In order to stabilize Two-wheeled vehicles in the state of stillness, center-of-gravity movement and handle operation by the rider are indispensable. Then we develop a stationary self-sustaining Two-wheeled vehicle which is a two-wheeled vehicle equipped with a cart system to move a center-of-gravity of the vehicle for stabilizing the system. We derive a state space model of system based on Lagrange method and identified model parameters by control experiments. A robust attitude controller is designed via H_∞ Loop Shaping Design Procedure (LSDP). Experimental results show an effectiveness of the derived mathematical model and the designed robust attitude controller compared with LQ controller.

Keywords: Self-sustaning Two-wheeled Vehicle, Modeling, Robust Attitude Control, H_∞LSDP

1. Introduction

Stability of Two-wheeled vehicles depends on their running speed. The running vehicle at high speed is stable but the vehicle in a state of stillness is unstable. Therefore, Two-wheeled vehicle has been researched considerably for a long time. In 1971, analysis of going straight stability with four degrees of freedom models of Two-wheeled vehicle is done by Sharp[1]. This has progressed theoretical analysis of Two-wheeled vehicle and stability analysis. The model of Sharp expressed a characteristic of Two-wheeled vehicle in a running, and it is a nonlinear model. However, handling becomes difficult from complexity of the model if we will build a control system with the model. The analysis of a basic characteristic of Two-wheeled vehicle that uses the linear theory was already reported[2)3)]. But, Two-wheeled running vehicle was treated in this research result.

On the other hand, as for the research about stabilization of Two-wheeled vehicle, there are two research results by Tanaka[4] and Kamata[5]. These researches target Two-wheeled vehicle that makes it run in low speed, and the handle operation is considered. However, in the state of stillness and the rider are not considered. In order to stabilize Two-wheeled vehicles in the state of stillness, center-of-gravity movement and handle operation by the rider are indispensable.

Then we develop a stationary self-sustaining Two-wheeled vehicle which is a two-wheeled vehicle equipped with a cart system to move a center-of-gravity of the vehicle for stabilizing the system. We derive a state space model of the system based on Lagrange method and identify the model parameters by control experiments. A robust attitude controller is designed via H_∞ Loop Shaping Design Procedure (LSDP). Experimental results show an effectiveness of the derived mathematical model and the designed robust attitude controller compared with LQ controller.

2. Composition and Modeling of an Experimental System

2.1 Composition of an Experimental System

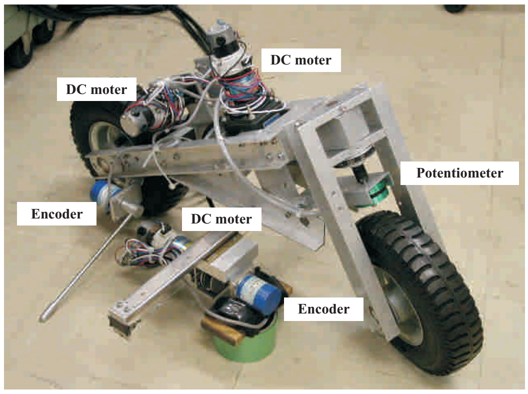

Figure 1: Overview of experimental system

Figure 1 is a photograph of the experimental system. The two-wheeled vehicle consists of three parts. There are a cart system that corresponds to the rider's center-of-gravity movement, a steering system (a front part) for steering, and a body (a rear part). the front part and the rear part are structures that finish being movable through a steering axis. A cart system and a steering system are driven by DC servo motor, and DC motors are controlled by servo amplifier which contains the velocity control system. Bike angle and cart position are

Table 1: Definition of Symbols

M_f, M_r, M_c	Mass of each part
H_f, H_r, H_c	Vertical length from a floor to a center-of-gravity of each part
L_{Ff}, L_F	Horizontal length from a front wheel rotation axis to a center-of-gravity of part of front wheel and steering axis.
L_r, L_R	Horizontal length from a rear wheel rotation axis to a center-of-gravity of part of rear wheel and steering axis.
L_c	Horizontal length from a rear wheel rotation axis to a center-of-gravity of the cart system.
J_x	Moment of inertia around center-of-gravity x axially.
J_{fz}	Moment of inertia for part of front wheel z axially.
J_z	Moment of inertia for part of rear wheel that contains cart system z axially.
μ_x	Viscous coefficient around x axis.
μ_{fz}	Viscous coefficient for part of front wheel around z axis.
μ_z	Viscous coefficient for part of rear wheel that contains cart system around z axis.
μ_c	A viscosity coefficient of a movement direction of the cart system
subscript f, r, c	Part of front wheel, rear wheel, and cart system respectively

measured by encoders. Handle angle is measured by a potentiometer.

A system's length is about 70 [cm], width is about 57 [cm], height is about 40 [cm], and weight is about 10 [kg]. Movable ranges of a cart system is ± 25 [cm], and a steering system is ± 0.5 [rad] respectively.

We used MATLAB, Simulink for a controller design and used dSPACE DSP-CIT for control experiments.

2.2 Preliminary

Figure 2 shows a model of Two-wheeled vehicle. We assume that Two-wheeled vehicle is stabilized by the cart movement $d(t)$ and the handle operation $\psi(t)$. The control inputs are the voltage $u_c(t)$, $u_h(t)$ to add to an amplifier. We assume that cart position $d(t)$, handle angle $\psi(t)$, and bike angle $\phi(t)$ can be measured directly.

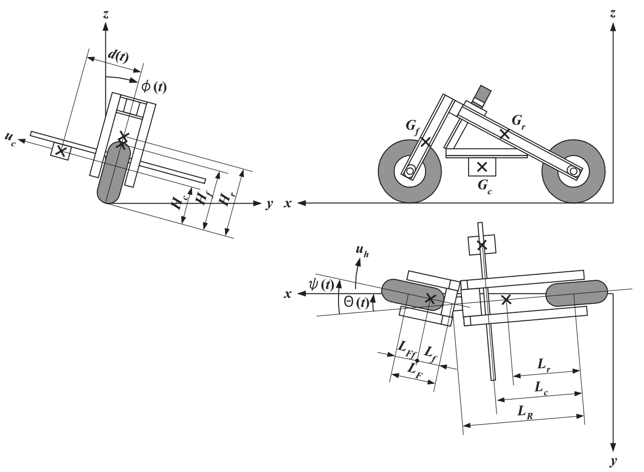

Figure 2: Two-wheeled vehicle model

For modeling, we consider the following assumptions.

1. The contact points on the ground of a front wheel and a rear wheel are set as the x axis, the y axis is orthogonal to the x axis, the z axis is vertical upward.
2. The bike angle $\phi(t)$, cart position $d(t)$, and handle angle $\psi(t)$ can be measured directly.
3. Two-wheeled vehicle is a structure that a front part and a rear part are connected with a steering axis.
4. The bike angle, cart position, and handle angle are small enough.
5. The center-of-gravity movement x axially and z axially by the handle operating are omitted.
6. The tire does not slip horizontally.
7. Two-wheeled vehicle is a rigid body, and the twist is not occurred.
8. A cart system and a steering system are driven by DC servo motor, and DC motors are controlled by servo amplifier which include the velocity control system.
9. The state variables which are differentiated twice are small enough. They can be omitted.

Table 1 shows the definition of the symbols in the expressions.

2.3 Yaw angle $\theta(t)$

Yaw angle $\theta(t)$ occurs between a rear part and the x axis by the handle operating. We cannot measure Yaw angle $\theta(t)$ directly. We can obtain it from the next expressions based on the geometric relations in Figure 3.

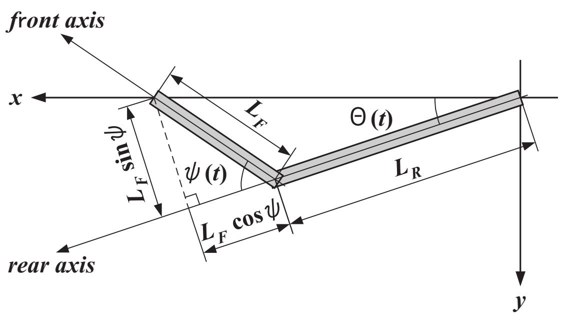

Figure 3: Relation between Steer angle $\psi(t)$ and Yaw angle $\theta(t)$

$$\tan\theta(t) = \frac{L_F \sin\psi(t)}{L_F \cos\psi(t) + L_R} \tag{1}$$

$$\begin{aligned}\theta(t) &= \tan^{-1}(A) \\ &= \sin^{-1}\frac{A}{\sqrt{1+A^2}} = \cos^{-1}\frac{1}{\sqrt{1+A^2}} \\ A &= \frac{L_F \sin\psi(t)}{L_R + L_F\cos\psi(t)}\end{aligned} \tag{2}$$

2.4 Center-of-gravity coordinates of a front part and a rear part

Figure 4 shows the center-of-gravity coordinates of the front part and the rear part. (a) and (b) in Figure 4 show the system with a turned handle, but Two-wheeled vehicle is not falling. (c) shows Two-wheeled vehicle is falling $\phi(t)$ from a state of (a) and (b).

When the handle is turned, Yaw angle $\theta(t)$ is occurred, and, the rear part has a few inclinational angle. Therefore, the center-of-gravity movement z axially is occurred. This variance is omitted from assumption, the center-of-gravity coordinates of a front part (y_f, z_f) and a rear part (y_r, z_r) are obtained as next expressions.

$$\begin{cases} y_f = H_f \sin\phi(t) + L_{Ff}\sin\{\psi(t)-\theta(t)\}\cos\phi(t) \\ z_f = H_f \cos\phi(t) - L_{Ff}\sin\{\psi(t)-\theta(t)\}\sin\phi(t) \end{cases} \tag{3}$$

$$\begin{cases} y_r = H_r \sin\phi(t) + L_r \sin\theta(t)\cos\phi(t) \\ z_r = H_r \cos\phi(t) - L_r \sin\theta(t)\sin\phi(t) \end{cases} \tag{4}$$

2.5 Center-of-gravity coordinate of a cart system

Figure 5 shows the center-of-gravity coordinates of a cart system. (a) and (b) in figure 5 show the system with the turned handle, but Two-wheeled vehicle is falling. (c) shows Two-wheeled vehicle is falling $\phi(t)$ from a state of (a) and (b).

When a handle is turned, the center-of-gravity movement z axially is occurred. But, this variance is omitted from the assumptions, the center-of-gravity coordinates of a cart system (y_c, z_c) is obtained by the next expressions.

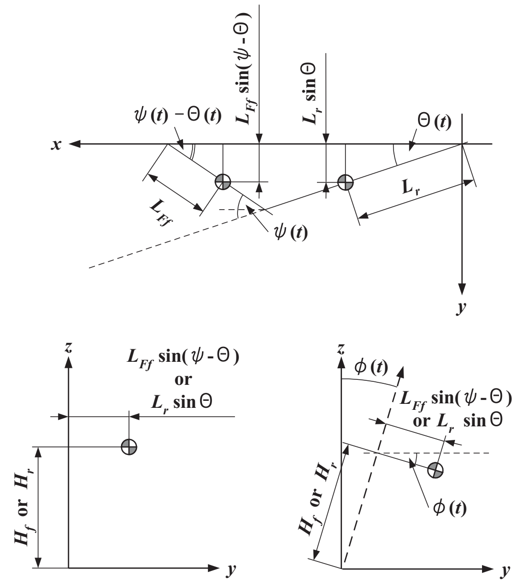

Figure 4: Center-of-gravity coordinates of a front part and a rear part

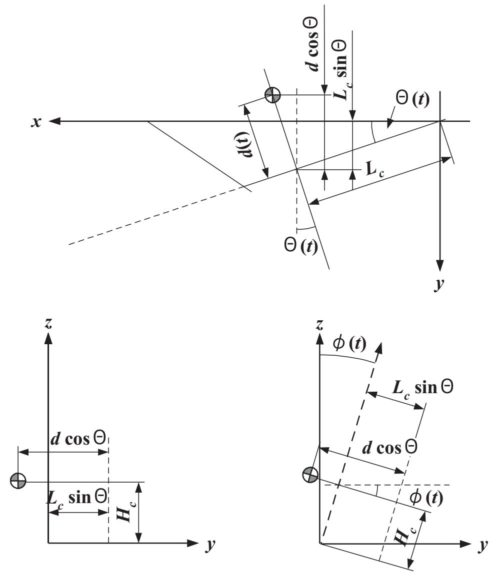

Figure 5: Center-of-gravity coordinates of cart

$$\begin{cases} y_c = H_c \sin\phi(t) + \{L_c\sin\theta(t) - d(t)\cos\theta(t)\}\cos\phi(t) \\ z_c = H_c \cos\phi(t) - \{L_c\sin\theta(t) - d(t)\cos\theta(t)\}\sin\phi(t) \end{cases} \tag{5}$$

2.6 Derivation of an equation of motion

From (3), (4) and (5), the motion energy T, the potential energy U, and the lost energy F are derived. Then, following expressions are obtained.

$$
\begin{aligned}
T &= \frac{1}{2}M_f\left(\dot{y_f}^2+\dot{z_f}^2\right)+\frac{1}{2}M_r\left(\dot{y_r}^2+\dot{z_r}^2\right) \\
&\quad +\frac{1}{2}M_c\left(\dot{y_c}^2+\dot{z_c}^2\right) \qquad (6)\\
&\quad +\frac{1}{2}\left\{J_x\dot{\phi}^2+J_z\left(\dot{\theta}\cos\phi\right)^2+J_{fz}\left(\dot{\psi}\cos\phi\right)^2\right\} \\
U &= g\left(M_f z_f+M_r z_r+M_c z_c\right) \qquad (7)\\
F &= \frac{1}{2}\mu_c\dot{d}^2+\frac{1}{2}\mu_x\dot{\phi}^2+\frac{1}{2}\mu_z\left(\dot{\theta}\cos\phi\right)^2 \\
&\quad +\frac{1}{2}\mu_{fz}\left(\dot{\psi}\cos\phi\right)^2 \qquad (8)
\end{aligned}
$$

Above expressions are substituted for Lagrange's motion equation described as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}+\frac{\partial F}{\partial \dot{q}_i}=\tau_i. \qquad (9)$$

But, from the assumptions, the motion equation of a cart system and a handle system are expressed by the following equations.

$$\begin{cases}\ddot{d}(t)+\alpha\dot{d}(t)=\beta u_c(t)\\ \ddot{\psi}(t)+\gamma\dot{\psi}(t)=\delta u_h(t)\end{cases} \qquad (10)$$

Where, α, β, δ, and γ are physical parameters of the motor systems.

Therefore, we solve it by the generalized coordinates $q_i=\phi(t)$ and the external force $\tau_i=0$. We use the Taylor expansion in the equilibrium ($d(t)=\phi(t)=\psi(t)=0$) neighborhood and the variable differentiated twice or more are omitted, the motion equation of next expression is obtained.

$$
\begin{aligned}
&\left\{J_x+M_fH_f^2+M_rH_r^2+M_cH_c^2\right\}\ddot{\phi}+M_fL_{Ff}H_f\ddot{\psi} \\
&\quad -M_cH_c\ddot{d}+\mu_x\dot{\phi}+gM_cd \\
&\quad -g(M_rL_r+M_cL_c-M_fL_{Ff})\psi\frac{L_F}{L_F+L_R} \\
&\quad -gM_fL_{Ff}\psi-g\phi(M_fH_f+M_rH_r+M_cH_c)=0
\end{aligned} \qquad (11)
$$

2.7 Derivation of the state space model

We substitute (10) into (11) and arrange it, we obtained the final state space linear model shown in the following expressions.

$$\begin{aligned}\dot{x}&=Ax+Bu\\ y&=Cx\end{aligned} \qquad (12)$$

where,

$$x=\left[\begin{array}{cccccc} d(t) & \phi(t) & \psi(t) & \dot{d}(t) & \dot{\phi}(t) & \dot{\psi}(t)\end{array}\right]^T$$

$$A=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&-\alpha&0&0\\a_{51}&a_{52}&a_{53}&a_{54}&a_{55}&a_{56}\\0&0&0&0&0&-\gamma\end{bmatrix}, B=\begin{bmatrix}0&0\\0&0\\0&0\\\beta&0\\b_{51}&b_{52}\\0&\delta\end{bmatrix}$$

$$C=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{bmatrix},\ u=\left[\begin{array}{cc}u_c(t) & u_h(t)\end{array}\right]^T$$

$$a_{51}=-\frac{M_cg}{den},\ a_{52}=\frac{(M_fH_f+M_rH_r+M_cH_c)\,g}{den},$$

$$a_{53}=\frac{\{M_rL_rL_F+M_cL_cL_F+M_fL_{Ff}L_R\}\,g}{(L_R+L_F)\,den},$$

$$a_{54}=-\frac{M_cH_c\alpha}{den},\ a_{55}=-\frac{\mu_x}{den},\ a_{56}=\frac{M_fH_fL_{Ff}\gamma}{den},$$

$$b_{51}=\frac{M_cH_c\beta}{den},\ b_{52}=-\frac{M_fH_fL_{Ff}\delta}{den}$$

$$den=M_fH_f^2+M_rH_r^2+M_cH_c^2+J_x$$

Unknown parameters in above expressions were identified by control experiments. Table 2 shows physical parameters of The Two-wheeled vehicle.

Table 2: Physical parameters of Two-wheeled vehicle

Parameter	Value	Parameter	Value
$M_f\ [kg]$	2.14	$H_f\ [m]$	0.0800
$M_r\ [kg]$	5.91	$H_r\ [m]$	0.161
$M_c\ [kg]$	1.74	$H_c\ [m]$	0.0980
$L_{Ff}\ [m]$	0.0390	$L_F\ [m]$	0.133
$L_r\ [m]$	0.128	$L_R\ [m]$	0.308
$L_c\ [m]$	0.259		
$J_x\ [kgm^2]$	0.2	$\mu_x\ [kgm^2/s]$	0.333
α	905	β	255
γ	98.96	δ	222.07

3. Controller design

3.1 H_∞ Loop Shaping Design Procedure

We used LSDP[6)7)] for the controller design. The open loop characteristic of the model is improved in LSDP with prepositive compensator W and a postpositive compensator V. Here, the prepositive compensator W was set to a frequency weight, and the postpositive compensator V was set to a constant weight.

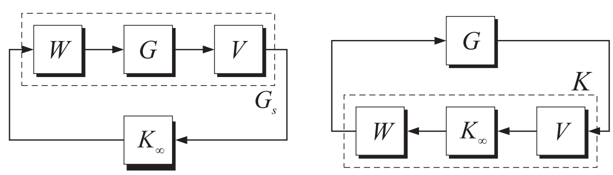

Figure 6: The loop shaping design procedure

$$
\begin{aligned}
W(s)&=diag\left(\left[\begin{array}{cc}W_c(s) & W_h(s)\end{array}\right]\right)\\
W_c(s)&=\frac{12.8}{s+10},\quad W_h(s)=\frac{1}{s+1}\\
V&=diag\left(\left[\begin{array}{ccc}40 & 40 & 40\end{array}\right]\right)
\end{aligned} \qquad (13)
$$

The controller was designed by using MATLAB. When (13) are used, we have $\gamma_{min}=13.3272$. Therefore, we used $\gamma=1.05\times\gamma_{min}$ for the controller design.

3.2 Optimal Regulator (LQ)

The optimal regulator is one of the state feedback control, and derives the feedback gain F of the control input $u = -Fx$ which minimize the following cost function.

$$J = \int_0^\infty \left(x^T Q x + u^T R u \right) dt \tag{14}$$

Where, weight $Q \geq 0$ and $R > 0$ were set respectively as

$$\begin{aligned} Q &= diag\left(\begin{bmatrix} 80 & 50 & 150 & 10 & 1 & 1 \end{bmatrix} \right) \\ R &= \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}. \end{aligned} \tag{15}$$

The all states are necessary to use the feedback gain by the optimal regulator method. However, we can get only one part ($d(t)$, $\phi(t)$, $\psi(t)$) of them. Then, we use the observer. Where, we use the pole placement method for designed the state observer. the observer poles were moved -25 from the regulator poles.

When we considered the system as a controller which is from model outputs $y(t)$ to operations $u(t)$ and this system is composed as shown in Figure 7. This system is called as LQ controller.

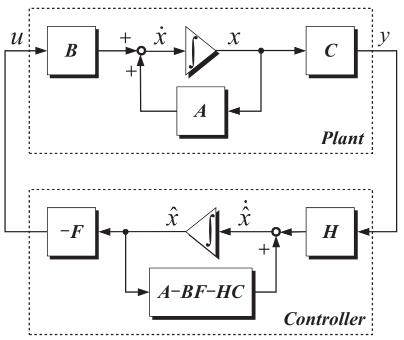

Figure 7: Configuration of LQ controller

3.3 Open Loop Characteristics

Figure 8, 9 show the open loop characteristic GK for the designed controllers.

The both controllers had a similar gain characteristics at the low frequency. However, LSDP controller has a low gain characteristic compared with LQ controller at the high frequency, and it means the controller should have robustness at the high frequency.

4. Attitude Control Experiments

The attitude control experiments are done by using the designed controllers. We experiment on the step responses and the impulse disturbance responses. Stabilization by the designed controllers was able to be achieved. As a result, the effectiveness of the independent self-sustaining two-wheeled vehicle and the derived two-wheeled vehicle model was able to be proven.

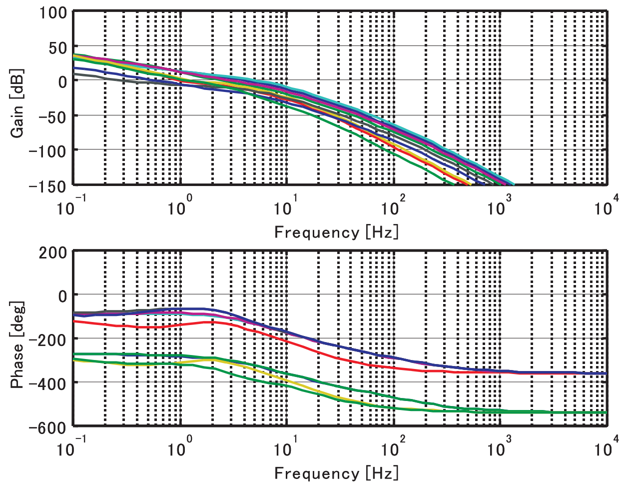

Figure 8: Frequency response of GK for LSDP controller

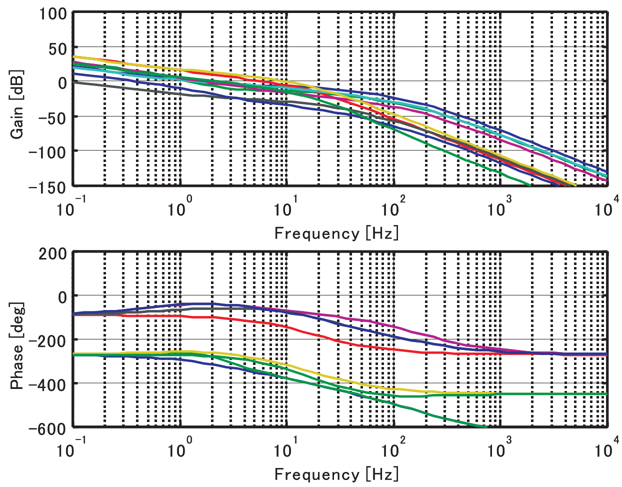

Figure 9: Frequency response of GK for LQ controller

4.1 Step Responses

-0.03 [m], -0.05 [rad] and 0.07 [rad] were given to the cart position, bike angle, and handle angle respectively as step references. Figure 10 shows the response results. The step value was inputted at 1 [sec] on the graphs.

The above graphs show cart position, bike angle, and handle angle respectively. The solid lines are LSDP controller's responses, and the dashed lines are LQ controller's responses.

When both responses are compared, the response of cart position and bike angle are similar. LSDP controller is only a little vibrating in all responses. However, both controllers' responses are corresponding to the step value, and stabilization can be achieved.

4.2 Impulse Disturbance Responses

Figure 11 shows the impulse disturbance response results. The impulse disturbance voltage of 10 [N] considerably is added to the operation voltage of the cart for 0.1 sec. As well as the step response, cart posi-

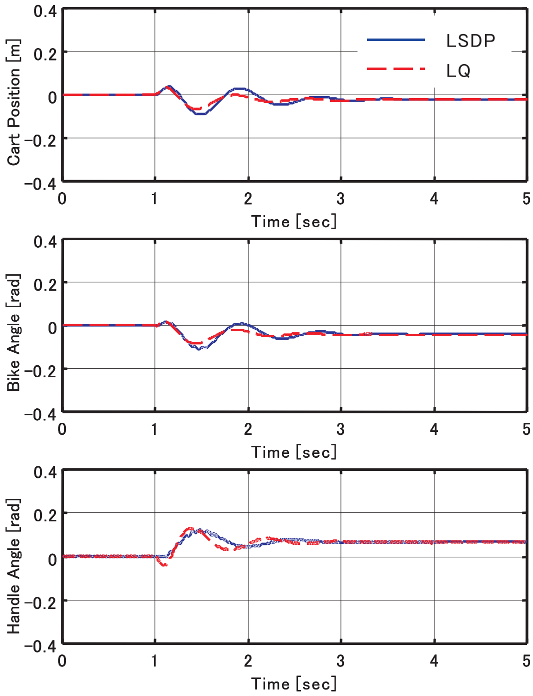

Figure 10: Step responses

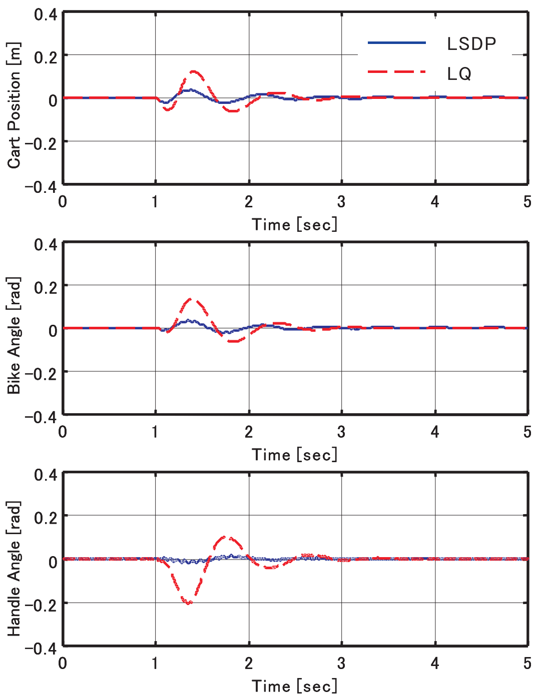

Figure 11: Impulse disturbance responses

tion, body angle and handle angle are shown in Figure 11 respectively. The solid lines are LSDP controller's responses, and the dashed lines are LQ controller's responses. The disturbance was inputted at 1 [sec] on the graphs.

In the step responses, the difference did not appear in the responses by both controllers. However, the difference appeared to both responses in the impulse disturbance responses. LSDP controller showed a better result in which the attitude change after disturbance was inputted compared with LQ controller. Especially, LQ controller has caused the attitude change. On the other hand, after disturbance was input, LSDP controller hardly operates the handle, and seems to control the large attitude change caused by the handle vibration. As a result, the LSDP controller has a better robustness.

5. Conclusions

In this paper, we developed a stationary self-sustaining Two-wheeled vehicle which is a two-wheeled vehicle equipped with a cart system to move a center-of-gravity of the vehicle for stabilizing the system. A motion equation of the model was derived by the Lagrange method. The two-wheeled vehicle model limited in the stationary was derived. The controllers were designed by LSDP and LQ, and both controllers stabilized Two-wheeled vehicle. In the impulse disturbance response, LSDP controller showed a better result. As a results, the effectiveness of the independent self-sustaining two-wheeled vehicle and the derived two-wheeled vehicle model was able to be proven. In the future, we will design a control system for the running two-wheeled vehicle and evaluate the effectiveness.

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