区間力学系の大偏差原理とレート関数の零点の構造について

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Consider a dynamical system $f\colon X\to X$ of a compact space X. The theory of large deviations deals with the behavior of the empirical mean

$$\delta_x^n=\frac{1}{n}\left(\delta_x+\delta_{f(x)}+\cdots+\delta_{f^{n-1}(x)}\right)\quad \text{as } n\to+\infty,$$

where δ_x denotes the Dirac measure at x. We put a Lebesgue measure $|\cdot|$ on X as a reference measure, and ask the asymptotic behavior of the empirical mean for Lebesgue almost every initial condition.

Let $\mathcal{M}$ denote the space of Borel probability measures on X endowed with the topology of weak convergence. We say *the Large Deviation Principle* (the LDP) holds if there exists a lower semi-continuous function $\mathscr{I}=\mathscr{I}(f;\cdot)\colon\mathcal{M}\to[0,+\infty]$ which satisfies the following:

- (lower bound) for every open subset $\mathcal{G}$ of $\mathcal{M}$,

$$\liminf_{n\to+\infty}\frac{1}{n}\log|\{x\in X\colon \delta_x^n\in\mathcal{G}\}|\geq-\inf_{\mu\in\mathcal{G}}\mathscr{I}(\mu);$$

- (upper bound) for every closed subset $\mathcal{K}$ of $\mathcal{M}$,

$$\limsup_{n\to+\infty}\frac{1}{n}\log|\{x\in X\colon \delta_x^n\in\mathcal{K}\}|\leq-\inf_{\mu\in\mathcal{K}}\mathscr{I}(\mu),$$

where $\log 0=-\infty$, $\inf\emptyset=\infty$ and $\sup\emptyset=-\infty$. The function $\mathscr{I}$ is called *a rate function*.

For a transitive uniformly hyperbolic system with Hölder continuous derivative, the LDP was established by Takahashi [6], Orey and Pelikan [5], Kifer [4], and Young [8]. For non-hyperbolic systems, few results on the LDP were available until recently. A substantial progress has been made in [1] in which the LDP was established for *every* multimodal map with non-flat critical point and Hölder continuous derivatives that is topologically exact. Our aim here is to establish the LDP for unimodal maps with non-recurrent flat critical point. We also study the structure of the set of zeros of the rate function for a concrete unimodal map.

In what follows, let $X=[0,1]$ and $f\colon X\to X$ be *a unimodal map*, i.e., a C^1 map whose critical set $\{x\in X\colon Df(x)=0\}$ consists of a single point $c\in(0,1)$ that is an extremum. We say f is *topologically exact* if for any open subset U of X there exists an integer $n\geq1$ such that $f^n(U)=X$ *An S-unimodal map* is a unimodal map of class C^3 on $X\setminus\{c\}$ with negative Schwarzian derivative. Let $\omega(c)$ denote the omega-limit set of c. The critical point c is *non-recurrent* if $c\notin\omega(c)$, and is *flat* if there exists a C^3 function ℓ on $X\setminus\{c\}$ such that:

(i) $\ell(x)\to+\infty$ and $|D\ell(x)|\to+\infty$. Here, $x\to c$ indicates both as $x\to c+0$ and $x\to c-0$;

(ii) there exist C^1 diffeomorphisms ξ, η of $\mathbb{R}$ such that $\xi(c)=0=\eta(f(c))$ and $|\xi(x)|^{\ell(x)}=\eta(f(x))$ for all x near c.

The flat critical point c is *of polynomial order* if there exists a C^3 function v on X such that $v(c)>0$ and for all x near c, $\ell(x)=|x-c|^{-v(x)}$. Define $\mathscr{F}=\mathscr{F}(f;\cdot)\colon\mathcal{M}\to[-\infty,0]$ by

$$\mathscr{F}(\nu)=\begin{cases}h(\nu)-\int\log|Df|d\nu & \text{if } \nu \text{ is } f\text{-invariant;}\\ -\infty & \text{otherwise.}\end{cases}$$

The $-\mathscr{F}$ is not lower semi-continuous. Hence, we introduce its lower semi-continuous regularization $\mathscr{I}=\mathscr{I}(f;\cdot)$ by

$$\mathscr{I}(\mu)=-\inf_{\mathcal{G}\ni\mu}\sup_{\nu\in\mathcal{G}}\mathscr{F}(\nu),$$

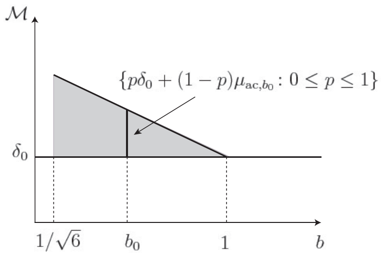

FIGURE 1. The sets of zeros of the rate functions for the family $\{f_b\}_{b>0}$.

where the infimum is taken over all open subsets $\mathcal{G}$ of $\mathcal{M}$ containing μ.

Theorem A. ([2]) *Let $f\colon X \to X$ be a topologically exact S-unimodal map with non-recurrent flat critical point that is of polynomial order. Then the LDP holds. The rate function is given by $\mathscr{I}$.*

We now consider a parametrized family $\{f_b\}_{b>0}$ of unimodal maps given by

$$f_b(x) = \begin{cases} -2^{2^b}\,|x-1/2|^{|x-1/2|^{-b}} + 1 & \text{for } x \in [0,1] \setminus \{1/2\}; \\ 1 & \text{for } x = 1/2. \end{cases}$$

The $1/2$ is a flat critical point of polynomial order. Theorem A applies to the map f_b. This map has an invariant measure that is absolutely continuous with respect to the Lebesgue measure. This measure is finite if and only if $b < 1$. In this case, the normalized measure is denoted by $\mu_{\mathrm{ac},b}$. We have a complete characterization of the zeros of the rate function for f_b:

Theorem B. ([2]) *The following holds for $\{f_b\}$:*

- *for $1/\sqrt{6} \le b < 1$, $\mathscr{I}(f_b;\mu) = 0$ if and only if there exists $p \in [0,1]$ such that $\mu = p\delta_0 + (1-p)\mu_{\mathrm{ac},b}$;*
- *for $b \ge 1$, $\mathscr{I}(f_b;\mu) = 0$ if and only if $\mu = \delta_0$.*

Combining the result [7, Theorem A.2] and that of Freitas and Todd [3] one can show that $b \in [1/\sqrt{6}, 1) \mapsto \mu_{\mathrm{ac},b} \in \mathcal{M}$ is continuous (continuous in the L^1 norm). Also, one can show that $\mu_{\mathrm{ac},b}$ converges weakly to δ_0 as $b \nearrow 1$. As a consequence, the set of zeros of the rate function for f_b depends continuously on $b > 0$ (See FIGURE 1).

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Multiray generalization of the arcsine laws for occupation times of infinite ergodic transformations

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In this talk, we consider a certain distributional convergence of occupation time ratios for ergodic transformations preserving an infinite measure. We give a general limit theorem which can be regarded as a multiray extension of the 2-ray results by Thaler [3] and Thaler–Zweimüler [4]. We also explain applications to interval maps with indifferent fixed points.

1 Multiray generalized arcsine laws

Let $N \geq 2$ be an integer. For $\alpha \in (0,1)$ and $\beta = (\beta_1, \dots, \beta_N) \in [0,1]^N$ with $\sum_{i=1}^N \beta_i = 1$, let $(Z_t^{(\alpha,\beta)})_{t\geq 0}$ be a *skew Bessel diffusion process*, starting at 0, of dimension $2-2\alpha \in (0,2)$ and with skewness parameter β on N rays which are all connected at 0. In the special case of $N = 2$ and $\alpha = \beta_1 = \beta_2 = 1/2$, this process is nothing else but a standard one-dimensional Brownian motion. Let us denote by $A_i^{(\alpha,\beta)}$ the occupation time of $(Z_t^{(\alpha,\beta)})_{t\geq 0}$ on i-th ray up to time 1 for $i = 1, \dots, N$. Barlow–Pitman–Yor [1] showed

$$\left(A_i^{(\alpha,\beta)}\right)_{i=1}^N \overset{\mathrm{d}}{=} \left(\frac{\xi_i}{\sum_{j=1}^N \xi_j}\right)_{i=1}^N,$$

where $\xi_1, \dots, \xi_d$ are $\mathbb{R}_+$-valued independent random variables with the one-sided α-stable distributions characterized by $\mathbb{E}[\exp(-\lambda\xi_i)] = \exp(-\beta_i\lambda^\alpha)$ for $\lambda > 0,\ i = 1, \dots, N$. In the special case of $\alpha = \beta_1 = 1/2$, the $A_1^{\alpha,\beta}$ is arcsine distributed.

2 Main results

Let $(X, \mathcal{B}, \mu)$ be a standard measurable space with a σ-finite measure such that $\mu(X) = \infty$, and let $T : (X, \mathcal{B}, \mu) \to (X, \mathcal{B}, \mu)$ be a conservative, ergodic, measure preserving transformation (which is abbreviated by *CEMPT*), i.e., $\mu T^{-1} = \mu$ and $\sum_{k\geq 0} \mathbb{1}_A(T^k x) = \infty$, μ-a.e.x, for any $A \in \mathcal{B}$ with $\mu(A) > 0$.

Assumption 2.1. The state space X is decomposed into $X = \sum_{i=1}^N X_i + Y$ for the *rays* $X_i \in \mathcal{B}$ with $\mu(X_i) = \infty$ $(i = 1, \dots, N)$ and the *junction* $Y \in \mathcal{B}$ with $\mu(Y) = 1$ such that, *when the orbit* $(T^k x)_{k\geq 0}$ *changes rays, it must visit the junction.*

We will denote by $H_n(x)$ the n-th hitting time of $(T^k x)_{k\ge 0}$ for Y. Set

$$\ell_i^{n+1}(x) := \max\{k \ge 1\,;\, T^{H_n+1}x, \dots, T^{H_n+k}x \in X_i\}, \quad x \in Y,$$
$$\ell^n := (\ell_1^n, \dots, \ell_N^n),$$

where $\max\emptyset = 0$. Note that ℓ_i^n is the *n-th X_i-side excursion length of* $(T^k x)_{k\ge 0}$ *from* Y, and the sequence $(\ell^n)_{n\ge 1}$ is stationary w.r.t. a probability measure $\mu_Y := \mu(\cdot \cap Y)$.

Assumption 2.2. The sequence $(\ell^n)_{n\ge 1}$ under μ_Y may be regarded to be i.i.d. in a certain asymptotical sense.

Set $S_{n,i}(x) := \sum_{k=0}^{n-1} \mathbb{1}_{X_i}(T^k x)$ for $n \ge 0,\ i = 1, \dots, N$. We now give our general limit theorem as follows.

Theorem 2.3 (S.–Yano [2])**.** *Let $\alpha \in (0,1)$ and $\beta = (\beta_1, \dots, \beta_N) \in [0,1]^N$ with $\sum_{i=1}^N \beta_i = 1$. Suppose that T is a CEMPT on $(X, \mathcal{B}, \mu)$ and that Assumptions 2.1 and 2.2 hold. We consider the following conditions:*

(i) *For each $i = 1, \dots, N$ and $\lambda > 0$,*

$$\lim_{r\to\infty} \frac{\mu_Y(\ell_i^1 > \lambda r)}{\mu_Y(|\ell^1| > r)} = \beta_i \lambda^{-\alpha}.$$

(ii) *For any probability measure $\nu \ll \mu$ on X,*

$$\left(n^{-1} S_{n,i}\right)_{i=1}^N \ \text{under } \nu \ \xrightarrow[n\to\infty]{\mathrm{d}} \ \left(A_i^{(\alpha,\beta)}\right)_{i=1}^N.$$

Then, (i) *implies* (ii)*. Furthermore, if $\beta \in [0,1)^N$, then* (ii) *implies* (i)*.*

The case $N = 2$ was due to [3] and [4]. The proofs in [3] and [4] were based on the moment method, which does not seem to be suitable for our multiray case. We adopt instead the double Laplace transform method, which was utilized in the study [1] of occupation times of diffusions on multiray.

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確率Fourier係数によるrandom関数の再構成について

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1.　序

Random関数(乱関数)が確率Fourier係数(SFC)から再構成できるか, という問題が[1]〜[6], [8]で論じられてきた. 乱関数がcausalな場合については[1], [2]で論じられている. 本講演では, 乱関数がnoncausalな場合を考える. SFCがOgawa積分で与えられた場合についての結果は[5], [6]で与えられている. [5]では, 絶対連続な乱関数の同定について, 確率Fourier変換([1], [2])の手法が採用されている. 本研究では, 確率Fourier変換とは異なる手法を用いて任意の有界変動過程及びそれから定まる確率微分がSFCで特定される事を得た. このことを用いて, 絶対連続なWiener汎関数はSkorokhod積分で与えられたSFCから復元できる事も得た. ここで, それぞれの過程の絶対値は, SFCを定めるBrown運動の値を用いずに復元される.

2.　設定

$(B_t)_{t\in[0,\infty)}$を確率空間$(\Omega,\mathcal{F},P)$上のBrown運動, $0<L<\infty$とし, $L=\infty$のとき$[0,L]$を$[0,\infty)$とみなす. $(e_i)_{i\in\mathbb{N}}$を$L^2([0,L];\mathbb{C})$のcompact support関数からなるCONSとする. Ogawa積分, Sobolev空間, Skorokhod積分をそれぞれ$\int_0^L d_{\mathrm{u}}B$, $\mathcal{L}_i^{r,2}$, $\int_0^L dB$と表す([7]の定義1,2,4を参照). (Noncausalな)乱関数a,bは$a\in L^2([0,L];\mathbb{R})$ a.s. $b\in L^2([0,L];\mathbb{C})$ a.s. を満たすとする.

定義 1 (乱関数の確率微分のSFC-O) 任意の$i\in\mathbb{N}$に対し, ae_iはOgawa積分可能であるとする. Ogawa積分$\int_0^L\cdot\, d_{\mathrm{u}}B$による確率微分

$$d_{\mathrm{u}}Y_t=a(t)\,d_{\mathrm{u}}B_t+b(t)\,dt\quad, t\in[0,L]$$

の$(e_i)_{i\in\mathbb{N}}$に関する確率Fourier係数(SFC-O) $(e_i,d_{\mathrm{u}}Y)$を次で定義する.

$$(e_i,d_{\mathrm{u}}Y):=\int_0^L e_i(t)\,d_{\mathrm{u}}Y_t=\int_0^L a(t)e_i(t)\,d_{\mathrm{u}}B_t+\int_0^L b(t)e_i(t)\,dt.$$

定義 2 (Noncausal Wiener汎関数の確率微分のSFC-S) 任意の$i\in\mathbb{N}$に対し, $ae_i\in\mathcal{L}_1^{1,2}$であるとする. Skorokhod積分$\int_0^L\cdot\, dB$による確率微分

$$dX_t=a(t)\,dB_t+b(t)\,dt\quad, t\in[0,L]$$

の$(e_i)_{i\in\mathbb{N}}$に関する確率Fourier係数(SFC-S) (e_i,dX)を次で定義する.

$$(e_i,dX):=\int_0^L e_i(t)\,dX_t=\int_0^L a(t)e_i(t)\,dB_t+\int_0^L b(t)e_i(t)\,dt.$$

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3. 主結果

定理 1 (有界変動過程の確率微分のSFC-Osによる同定) a を可測有界変動過程とする. a, b は確率微分 $d_{\mathrm{u}}Y_t = a(t)\,d_{\mathrm{u}}B_t + b(t)\,dt$ の $(e_i)_{i\in\mathbb{N}}$ に関するSFC-Osの系 $((e_i, d_{\mathrm{u}}Y))_{i\in\mathbb{N}}$ で構成できる.

注1) a が概左連続でもあれば, a.s. で $a \in C[0,L]$ が構成できる.
注2) 有限個の $(e_i, d_{\mathrm{u}}Y)$ を除いても a は構成できる.
注3) $|a|$ の構成の過程で $(B_t)_{t\in[0,L]}$ を要しない.

命題 1 (Hilbert-Schmidt 積分表示された Wiener 汎関数の Ogawa 積分) $K \in L^2([0,L]^2), f \in \mathcal{L}_1^{1,2}$ とするとき,

$$F(t) := \int_0^L K(t,s)f(s)\,ds$$

はu-可積分で, その Ogawa 積分は次で与えられる.

$$\int_0^L F(t)\,d_{\mathrm{u}}B_t = \int_0^L F(t)\,dB_t + \int_0^L\int_0^L K(t,s)D_tf(s)\,ds\,dt \quad \text{in } L^2(\Omega).$$

系 1 (絶対連続な Wiener 汎関数の Ogawa 積分) a を概絶対連続で, $a' \in \mathcal{L}_1^{1,2}, a(0) \in \mathcal{L}_0^{1,2}$ とする. また, $e \in L^2[0,L]$ を compact support とする. このとき, ae の Ogawa 積分は次のように表示できる.

$$\int_0^L ae(t)\,d_{\mathrm{u}}B_t = \int_0^L ae(t)\,dB_t + \int_0^L \Big(\int_0^t D_ta'(s)\,ds + D_ta(0)\Big)e(t)\,dt \quad \text{in } L^2(\Omega) \text{ and a.s.}$$

定理 2 (絶対連続な Wiener 汎関数の確率微分の SFC-Ss による同定) a を概絶対連続で $a' \in \mathcal{L}_1^{1,2}, a(0) \in \mathcal{L}_0^{1,2}$ とする. このとき, a.s. で $a \in C[0,L]$ と b は確率微分 $dX_t = a(t)\,dB_t + b(t)\,dt$ の $(e_i)_{i\in\mathbb{N}}$ に関する SFC-Ss の系 $((e_i, dX))_{i\in\mathbb{N}}$ により構成できる.

注1) 有限個の (e_i, dX) を除いても a は構成できる.
注2) $|a|$ の構成の過程で $(B_t)_{t\in[0,L]}$ を要しない.

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一般 CONS の確率フーリエ係数による乱関数の復元について*

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(i) 確率 Fourier 係数.　$(\Omega, \mathcal{F}, P)$ を確率空間，$W.$ を $(\Omega, \mathcal{F}, P)$ 上の Brown 運動，$f(t,\omega)$ を $[0,1]\times\Omega$ 上のランダム関数とする。$L^2([0,1],dt)$ の完全正規直交基底 $\{\varphi_n(t)\}$ に対して，

$$\hat{f}_n(\omega) := \int_0^1 f(t,\omega)\overline{\varphi_n(t)}dW_t \tag{1}$$

を $f(t,\omega)$ の $\{\varphi_n(t)\}$ に対する確率 Fourier 係数 (SFC) と呼ぶ。ここで $\bar{z}$ は z の複素共役を表す。SFC (1) は，CONS $\{\varphi_n(t)\}$ および確率積分 $dW.$ の選択により定まることに注意する。我々の目的は $\{\hat{f}_n(\omega)\}$ から元の関数 $f(t,\omega)$ を再構成することにある。再構成において SFCs 以外の情報を用いないとき「強い意味での」再構成，用いる場合には「広い意味での」再構成という。今回の発表では確率積分として Ogawa 積分を選択し，CONS としては特定のものを仮定しない。本講ではまず Ogawa 積分について些かの一般論を展開する。その後 SFC と H^1 基底による Ogawa 積分の表現について説明したのち，強い意味および広い意味での再構成問題を考察する。

(ii) Ogawa 積分.　CONS $\{\psi_n(t)\}$ に対して $\sum \int_0^1 f(t,\omega)\overline{\psi_n(t)}dt \int_0^1 \psi_n(t)dW_t$ が確率収束するとき，$f(t,\omega)$ は $\{\psi_n(t)\}$ に関して Ogawa 積分可能であるといい，その和を $\{\psi_n(t)\}$-Ogawa 積分とよび $\int_0^1 f(t,\omega)d_\psi W_t$ と記す。

$\{\psi_n(t), n\in\mathbb{N}\}$, $\{\chi_n(t), n\in\mathbb{N}\}$ をそれぞれ $L^2([0,1],dt)$ の CONS とする。ランダム関数 $f(t,\omega)$ と $L^2([0,1],dt)$ 関数 $\alpha(t)$ に次の仮定をおく。

(O1)　$\exists\{\lambda_n\}$ s.t. $\lambda_n>0$, $\sum\lambda_n<\infty$,　$E\sum\frac{1}{\lambda_n}\left|\int_0^1 f(t,\omega)\overline{\chi_n(t)}dt\right|^2<\infty$

(O2)　$\alpha(t)\chi_\ell(t)\in L^2([0,1],dt)$　&　$\sup_\ell\int_0^1|\alpha(t)\chi_\ell(t)|^2dt<\infty$

(O3)　$\alpha(t)\psi_k(t)\in L^2([0,1],dt)$

命題 1. *(O1), (O2), (O3)* の下で

$$\int_0^1 f(t,\omega)\alpha(t)d_\psi W_t = \sum_{\ell=1}^\infty \int_0^1 f(t,\omega)\overline{\chi_\ell(t)}dt\int_0^1\alpha(t)\chi_\ell(t)dW_t$$

が成り立つ。

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注意 1. $\alpha(t)=1$ とすると $\int_0^1 f(t,\omega)d_\psi W_t = \int_0^1 f(t,\omega)d_\chi W_t$ を得る。すなわち *(O1)* は $f(t,\omega)$ の *Ogawa* 積分が *CONS* の選び方に依らないこと（u-可積分性）を保証する。

注意 2. $\alpha(t)=1_{[0,s]}(t)$ とすると

$$\int_0^s f(t,\omega)d_\psi W_t = \sum_{\ell=1}^\infty \int_0^1 f(t,\omega)\overline{\chi_\ell(t)}dt \int_0^s \chi_\ell(t)dW_t$$

(iii) Ogawa 積分の２次共変分.　ランダム関数 $g(t,\omega)$ と CONS $\{\eta_n(t), n\in\mathbb{N}\}$, $\{\chi_n(t), n\in\mathbb{N}\}$ に対し次を仮定する。

(O1')　$\exists\{\mu_n\}$ s.t. $\mu_n>0$, $\sum\mu_n<\infty$,　$E\sum\frac{1}{\mu_n}\left|\int_0^1 g(t,\omega)\overline{\eta_n(t)}dt\right|^2<\infty$

(O4)　$\sup_n \sup_{t\in[0,1]}|\chi_n(t)|=M<\infty$

$\Delta_n=\{0=t_0<t_1<\cdots<t_n=t\}$ とおく。

命題 2. *(O1), (O1'), (O4),* $\sum_n|\Delta_n|<\infty$ の下で

$$\lim_{|\Delta_n|\to 0}\sum_{\Delta_n}\int_{t_i}^{t_{i+1}} f(s,\omega)d_\chi W_s \int_{t_i}^{t_{i+1}} g(s,\omega)d_\eta W_s = \int_0^t f(s,\omega)g(s,\omega)ds$$

が *almost sure* 収束の意味で成り立つ。

(iv) SFC と H^1 基底による Ogawa 積分の表現.

$$\hat{f}_n(\omega) := \int_0^1 f(t,\omega)\overline{\varphi_n(t)}d_\psi W_t$$

で SFC を定義する。（次の命題 3 の仮定の下で well-defined である）

命題 3. *(O1), (O3), (O4)* の下で，任意の $t\in[0,1]$ *(fixed)* に対して

$$\sum_{n=1}^\infty \hat{f}_n(\omega)\int_0^t \varphi_n(s)ds = \int_0^t f(s,\omega)d_\psi W_s \quad in\ L^1(dP)$$

が成り立つ。

(v) $f(t,\omega)$ の再構成.

定理 1. $f(t,\omega)$ の値域が $\{z=x+iy : y>0\}\cup\{x+i0 : x\geq 0\}$ に含まれるとする。*(O1), (O3), (O4)* の下で $f(t,\omega)$ は $\{\psi_n(t)\}$-*Ogawa* 積分による *SFCs* から強い意味で復元される。

定理 2. *(O1), (O3), (O4)* の下で $f(t,\omega)$ は $\{\psi_n(t)\}$-*Ogawa* 積分による *SFCs* から広い意味で復元される。

ゲーム理論から見た非加法的測度：若干のコメント

河野 敬雄 (Norio KÔNO)

1. ゲーム理論について

本発表でいう「ゲーム理論」とはいわゆる「協力ゲーム理論」のことである．私の理解に従えば,「ゲーム理論」という場合，ノイマンに始まる協力ゲーム理論とナッシュに始まる非協力ゲーム理論とに大別される．協力ゲーム理論はさらに譲渡可能な協力ゲーム理論 (Transferable utility game, TU game あるいは side payment のある提携形ゲーム) とそうでない協力ゲーム理論 (Non Transferable utility game, NTU game) とにわかれる．TU game はプレイヤーの集合 N(有限集合) と N の部分集合の全体 2^N 上で定義された実数値関数 v で表現される．v は特性関数と呼ばれる．以下，特性関数形ゲーム (N, v) という．通常，特性関数形ゲームで要求される仮定は

(1) $v(\emptyset) = 0$, (2) $\forall S, T \in 2^N\ s.t.\ S \cap T = \emptyset;\ v(S) + v(T) \leq v(S \cup T)$ (supper additivity)

の 2 つである．さらに，ノイマン・モルゲンシュテルン (1944) ではいわゆるゼロサムゲームであること,

(3) $\forall S \in 2^N;\ v(S) + v(S^c) = 0$

を仮定している．

従って，上記の (1),(2),(3) を仮定すると，(4) 非負性 $v(S) \geq 0$ は仮定できない．しかし，(1) と (2) の優加法性と (4) の非負性から (5) 単調性 $S \subset T \Longrightarrow v(S) \leq v(T)$ が従う．私見だが (3) のゼロサム性は本質的な仮定ではないと思われるので以後は (1) と (5) のみを仮定したい．

ところで，協力ゲーム理論における基本概念を幾つか紹介しておく．有限集合 A の要素の数を $\sharp A$ で表す．

· 提携（結託 coalition）: 2^N の要素，つまりプレイヤーの部分集合のこと．

· 配分（imputaion）: $\sharp N$ 次元ベクトル $\vec{x} = (x_1, \ldots, x_{\sharp N})$ のこと．

協力ゲーム理論は何を研究対象にするのだろうか？簡単にいうと一定の合理性の仮定（個人合理性，全体合理性）のもとに一定の条件を満たす配分を求めること．解概念 (solution concept) という．解概念としては，安定集合，コア，仁，等いろいろあり，優劣は付けがたい．存在，一意性も保証されない．

· シャープレイ値 : Shapley(1953) によって導入された．(N, v) に対して一意に定義される「配分」のこと．

ノイマン・モルゲンシュテルンの協力ゲーム (N, v) ではゼロサムを仮定するから当然 v の非負性は仮定できない．しかし，simple game（単純ゲーム，または投票ゲーム）と言われる 0 と 1 にしか値を採らない文字通りシンプルなゲームは本発表で議論する非加法的測度の最も単純な例なのでもう少し詳しく紹介する (鈴木・武藤:1985. 『協力ゲームの理論』東京大学出版会, 第 8 章)．単純ゲームは勝利提携の全体 $\mathcal{W} \equiv \{A \in 2^N;\ v(A) = 1\}$ あるいは敗北提携の全体 $\mathcal{L} \equiv \{A \in 2^N;\ v(A) = 0\}$ によって完全に特徴づけられる．さらに，$\mathcal{W}, \mathcal{L}$ を用いて,「妨害提携」,「最小勝利提携」,「拒否権」,「独裁者」，単純ゲーム固有の性質（分類）として,「プロパー」,「強い」,「弱い」単純ゲーム等が定義される．さらに単純ゲームの範囲内でゲームの合成，和，積が定義出来る．これらの概念，性質はプレイヤー集合が無限集合

であっても殆どの場合定義でき，かつ，非加法的測度の最もシンプルな例となっている．

2.　非加法的測度

本発表では河邊 (2016, 非加法的測度と非線形積分.『数学』68 巻 3 号, 266-292) に従って，非加法的測度を次のように定義する．

定義 2.1. $(X, \mathcal{F})$ を可測空間とする．ここで，X は空でない抽象集合，$\mathcal{F}$ は各点 $x \in X$ を含む σ-algebra とする．このとき，次の 3 つの条件を満たす $\mathcal{F}$ 上で定義された関数を**非加法的測度**と呼び，$(X, \mathcal{F}, v)$ と表記する．

(1-1) $v : \mathcal{F} \longrightarrow [0, \infty]$（非負性）
(1-2) $v(\emptyset) = 0$ （下方有界性)
(1-3) $\forall A, B : A \subset B \Longrightarrow v(A) \leq v(B)$ （単調性）

しかしながら，定義の条件をこのように緩めてしまうと，広いクラスの非加法的測度一般に対して成立する深い内容の定理を得ることが難しくなる．さらなる付加的条件を課す必要性が生じる．河邊 (*ibid.*) には非加法的測度を特徴づける 30 以上の概念が列挙されている．その場合，当然それらの概念の相互関係，同値であるか，独立な概念であるか，包含関係にあるのか，等が問題になる．それらの関係性を理解する最も手っ取り早い方法は具体的な例を示すことである．

本発表の目的は，非加法的測度一般をいきなり考察するのは「しんどい」ので，まず手がかりとして協力ゲーム理論ではよく知られている単純ゲームの一般化である**単純非加法的測度**を定義して，いくつかの例を紹介することである．協力ゲーム理論で知られている概念や定理がどこまで非加法的測度の研究に有効であるかは今後の課題としたい．

定義 2.2. 次の条件を満たす非加法的測度 $(X, \mathcal{F}, v)$ を単純非加法的測度という．

$$\forall A \in \mathcal{F},\ v(A) = 0 \text{ or } 1.$$

値が 0 と 1 しか取らない加法的測度は単位分布しかないが単純ゲームを例として考えただけでも単純非加法的測度は驚くほどの多様性を持っていることがわかる．

プレイヤーの数（以下，プレイヤーの集合＝可測空間 X である）が無限集合の場合に重要となる概念のひとつに連続性の問題がある．N は自然数の集合，A_n は $\mathcal{F}$ に属する集合の列 $n \in N$ とする．

(1) 順序連続性：$\forall A_n \downarrow \emptyset \Longrightarrow v(A_n) \downarrow 0.$
(2) 強順序連続性：$\forall A, A_n \downarrow A, v(A) = 0 \Longrightarrow v(A_n) \downarrow 0.$
(3) 上からの連続性：$\forall A, A_n \downarrow A \Longrightarrow v(A_n) \downarrow v(A).$

河邊では定義されていないが，非加法的測度では $v(A) + v(A^c) = v(X)$ が成り立つとは限らないから下からの連続性も細かく分類する必要がある．これらの概念は単純非加法的測度の範囲でも例を作ることができる．さらに単純非加法的測度の合成，和，積についても紹介する．

文献： Grabisch, M., 2016. *Set Functions, Games and Capacities in Decision Making.* Theory and Decision Library C. Game Theory, Social Choice, Decision Theory, and Optimization Volmume 46. Springer.

Existence and uniqueness results for one type of first order conservation laws involving a Q-Brownian motion

by

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Abstract :

We consider a first order conservation law with a multiplicative source term involving a Q-Brownian motion. We first present the result that the discrete solution obtained by a finite volume method converges along a subsequence in the sense of Young measures to a measure-valued entropy solution as the maximum diameter of the volume elements and the time step tend to zero. This convergence result yields the existence of a measure-valued entropy solution.

We then prove the uniqueness of the measure-valued entropy solution. We present the Kato inequality and as a corollary we deduce the uniqueness result. The Kato inequality is proved by a doubling of variables method; to that purpose, we prove the existence and the uniqueness of the weak solution of an associated nonlinear parabolic problem.

In the proof of the associated nonlinear parabolic problem, we apply an implicit time discretization to obtain a semi-discrete solution and prove the convergence of the discrete solution by using Itô's formula and a priori estimates. The convergence result yields the existence of a weak solution and we then prove the uniqueness of the weak solution.

Finally we show some numerical results for stochastic Burgers equation.

This is joint work with Tadahisa Funaki and Danielle Hilhorst.

A RELATION BETWEEN MODELED DISTRIBUTIONS AND PARACONTROLLED DISTRIBUTIONS

MASATO HOSHINO (WASEDA UNIVERSITY)

In the field of singular SPDEs, there are two big theories: the theory of *regularity structures* [4] by Hairer and the *paracontrolled calculus* [2] by Gubinelli, Imkeller and Perkowski. These two theories are based on a common principle but composed of different mathematical tools. Therefore we can use either of them according to the situation. For example, the former is useful to show a universal property of a large number of SPDEs (e.g. [5, 6]), and the latter is useful to get more detailed information of a specific SPDE (e.g. [3, 7]). However, there is a gap between the two theories about the range of application. For example, the Hairer's theory can be applied to the 3-dimensional parabolic Anderson model

$$(\partial_t - \Delta)u(t,x) = u(t,x)\xi(x), \quad t > 0,\ x \in \mathbb{T}^3,$$

for $\xi \in \mathcal{C}^{-3/2-\epsilon}(\mathbb{T}^3)$ with $\epsilon > 0$, but the GIP theory cannot be.

In this talk, we discuss how to overcome this gap. Recently, Bailleul and Bernicot [1] are tying to improve the GIP theory. Our plan is to complete their work by combining the essence of the Hairer's theory. There is a difference between both theories about the definition of solutions. In the Hairer's theory, the solution is defined as a *modeled distribution*, which represents a local behavior of the solution. In the GIP theory, the solution is defined as a *paracontrolled distribution*, which is defined by nonlocal operators. Each definition has an advantage to each other. We compare these two notions and aim to find a better way.

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On the Gibbs equilibrium in stochastic complex Ginzburg-Landau equations

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Abstract
In Physics, the stochastic Gross-Pitaevskii equation is used as a model to describe Bose-Einstein condensation at positive temperature. The equation is in fact a complex Ginzburg-Landau equation with a trapping potential and an additive space-time white noise. I am going to talk about two important questions and corresponding our results for this system: the global existence of solutions in the support of the Gibbs measure, and the convergence of those solutions to the equilibrium for large time. This is a joint work with A. de Bouard (Ecole Polytechnique) and A. Debussche (ENS Rennes).

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The invariant measure and flow associated to the Φ^4_3-quantum field model

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We consider the invariant measure and flow for the stochastic quantization equation associated to the Φ^4_3-model on the torus, which appears in quantum field theory. By virtue of Hairer's breakthrough, such nonlinear stochastic partial differential equations became solvable and are intensively studied now. In this talk, we present a direct construction to both a global solution and an invariant measure for this equation.

Let $m_0 > 0$, Λ be the 3-dimensional torus i.e. $\Lambda = (\mathbb{R}/\mathbb{Z})^3$, and μ_0 be the centered Gaussian measure on the space of Schwartz distributions $\mathcal{S}'(\Lambda)$ with the covariance operator $[2(-\triangle+m_0^2)]^{-1}$. We remark that μ_0 is different from the Nelson's Euclidean free field measure by the scaling $\sqrt{2}$. In order to adjust our setting to those of known results, we define μ_0 as above. In the constructive quantum field theory, there was a problem to construct a measure

$$\mu(d\phi) = Z^{-1} \exp\left(-U(\phi)\right) \mu_0(d\phi)$$

where

$$U(\phi) = \int_\Lambda \left(\frac{\lambda}{4}\phi(x)^4 - C_\lambda \phi(x)^2 \right) dx,$$

$\lambda > 0$ and Z is the normalizing constant. Since the support of μ_0 is in the space of tempered distributions, ϕ^4 and ϕ^2 are not defined in usual sense. So, we approximate ϕ and take the limit.

Let $\langle f, g\rangle$ be the inner product on $L^2(\Lambda;\mathbb{C})$. For $k \in \mathbb{Z}^d$, define $e_k(x) := e^{2\pi i k\cdot x}$ where $k\cdot x := k_1x_1 + k_2x_2 + k_3x_3$. For $N \in \mathbb{N}$, denote $\{j \in \mathbb{Z}; |j| \le N\}$ by $\mathbb{Z}_N$, and let P_N be the mapping from $\mathcal{S}'(\Lambda)$ to $L^2(\Lambda;\mathbb{C})$ given by

$$P_N\phi := \sum_{k\in\mathbb{Z}_N^3} \langle \phi, e_k\rangle e_k.$$

Define a function U_N on $\mathcal{S}'(\Lambda)$ by

$$U_N(\phi) = \int_\Lambda \left\{ \frac{\lambda}{4}(P_N\phi)(x)^4 - \frac{3\lambda}{2}\left(C_1^{(N)} - 3\lambda C_2^{(N)}\right)(P_N\phi)(x)^2 \right\} dx$$

where

$$C_1^{(N)} = \frac{1}{2}\sum_{k\in\mathbb{Z}_N^3} \frac{1}{k^2+m_0^2}, \quad C_2^{(N)} = \frac{1}{2}\sum_{l_1,l_2\in\mathbb{Z}_N^3} \frac{1}{(l_1^2+m_0^2)(l_2^2+m_0^2)(l_1^2+l_2^2+(l_1+l_2)^2+3m_0^2)}.$$

We remark that $\lim_{N\to\infty} C_1^{(N)} = \lim_{N\to\infty} C_2^{(N)} = \infty$, and $C_1^{(N)}$ and $C_2^{(N)}$ are called renormalization constants. Consider the probability measure μ_N on $\mathcal{S}'(\Lambda)$ given by

$$\mu_N(d\phi) = Z_N^{-1} \exp\left(-U_N(\phi)\right) \mu_0(d\phi)$$

where Z_N is the normalizing constant. We remark that $\{\mu_N\}$ is the approximation sequence of the Φ_3^4-measure which will be constructed below as the invariant measure of the associated flow.

Now we consider the stochastic quantization equation associated to $\{\mu_N\}$ as follows.

$$\begin{cases} d\tilde{X}_t^N(x) &= dW_t(x) - (-\triangle + m_0^2)\tilde{X}_t^N(x)dt \\ & \qquad -\lambda \left\{ P_N[(P_N\tilde{X}_t^N)^3](x) - 3\left(C_1^{(N)} - 3\lambda C_2^{(N)}\right) P_N\tilde{X}_t^N(x) \right\} dt \\ \tilde{X}_0^N(x) &= \xi_N(x) \end{cases}$$

where $W_t(x)$ is a white noise with parameter $(t,x) \in [0,\infty) \times \Lambda$ and $\xi_N(x)$ is a random variable which has μ_N as the law, and independent of W_t. We remark that μ_N is the invariant measure with respect to the semigroup generated by the solution to the equation. Let $X^N := P_N\tilde{X}^N$ for $N \in \mathbb{N}$. Then, X^N satisfies the stochastic partial differential equation

$$\begin{cases} dX_t^N(x) &= P_N dW_t(x) - (-\triangle + m_0^2)X_t^N(x)dt \\ & \qquad -\lambda \left\{ P_N[(X_t^N)^3](x) - 3(C_1^{(N)} - 3\lambda C_2^{(N)})X_t^N(x) \right\} dt \\ X_0^N(x) &= P_N\xi_N(x) \end{cases} \tag{1}$$

To apply the Hairer's reconstruction method, which enables us to transform (1) for a solvable partial differential equation, we supplementary introduce the infinite-dimensional Ornstein-Uhlenbeck process Z as follows. Let Z be the solution to the stochastic partial differential equation on Λ

$$\begin{cases} dZ_t(x) &= dW_t(x) - (-\triangle + m_0^2)Z_t(x)dt, \quad (t,x) \in [0,\infty) \times \Lambda \\ Z_0(x) &= \zeta(x), \qquad\qquad\qquad\qquad\qquad x \in \Lambda \end{cases}$$

where ζ is a random variable which has μ_0 as its law and is independent of W_t and ξ_N. Let $X_t^{N,(2)} := X_t^N - \mathcal{Z}_t^{(1,N)} + \lambda \mathcal{Z}_t^{(0,3,N)}$ for $t \in [0,\infty)$ where

$$\mathcal{Z}_t^{(0,3,N)} := \int_0^t e^{(t-s)(\triangle - m_0^2)} \left(P_N(P_N Z_s)^3 - 3C_1^N P_N Z_s \right) ds, \quad t \in [0,\infty),$$

and decompose $X^{N,(2)}$ into $X^{N,(2),<}$ and $X^{N,(2),\geqslant}$ by means of paraproduct. Then, we have a solvable, coupled, semilinear and dissipative parabolic partial differential equation, which the pair $(X^{N,(2),<}, X^{N,(2),\geqslant})$ satisfies. By applying the technique of the semilinear and dissipative parabolic equation, we obtain some estimates for $X^{N,(2),<}$ and $X^{N,(2),\geqslant}$, which yields the tightness of $X^{N,(2)}$. As the result we obtain the following theorem for the Φ_3^4-measure and the associated flow.

Theorem 1. *For $\varepsilon \in (0,1]$ sufficiently small, $\{X^N\}$ is tight on $C([0,\infty); B_{4/3}^{-1/2-\varepsilon})$, where $B_{p,r}^s$ is the Besov space. Moreover, if X is a limit of a subsequence $\{X^{N(k)}\}$ of $\{X^N\}$ on $C([0,\infty); B_{4/3}^{-1/2-\varepsilon})$, then X is a continuous Markov process on $B_{4/3}^{-1/2-\varepsilon}$, the limit measure μ of the associated subsequence $\{\mu_{N(k)}\}$ is an invariant measure with respect to X and it holds that*

$$\int \|\phi\|_{B_4^{-1/2-\varepsilon}}^4 \mu(d\phi) < \infty.$$